OXFORD BULLETIN OF ECONOMICS AND STATISTICS, 75, 2 (2013) 0305-9049
doi: 10.1111/j.1468-0084.2011.00683.x

Quantile Autoregressive Distributed Lag Model with an Application to House Price Returns*

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Abstract

This article studies quantile regression in an autoregressive dynamic framework with exogenous stationary covariates. We demonstrate the potential of the quantile autoregressive distributed lag model with an application to house price returns in the United Kingdom. The results show that house price returns present a heterogeneous autoregressive behaviour across the quantiles. Real GDP growth and interest rates also have an asymmetric impact on house prices variations.

I. Introduction

Asymmetric dynamic responses are common in the time series empirical literature. For instance, Beaudry and Koop (1993) show that positive shocks to the US GDP are more persistent than negative shocks. Poterba (1991) and Capozza *et al.* (2002) among others, present evidence on the asymmetric responses of house prices to income shocks. The occurrence of these asymmetries call into question the usefulness of models with time invariant structures as means of modelling such series. Quantile regression (QR) is a statistical method for estimating models of conditional quantile functions, which offers a systematic strategy for examining how covariates influence the location, scale and shape of the entire response distribution, therefore exposing a variety of heterogeneity in response dynamics. Koenker and Xiao (2006) introduced quantile autoregression (QAR) models in which the autoregressive coefficients can be expressed as monotone functions of a single, scalar random variable. QAR models are becoming increasingly popular, and there is a growing

*The authors would like to express their appreciation to Christopher Adam, two anonymous referees, Dan Bernhardt, Odilon Camara, Roger Koenker, Luiz Lima, Simone Manganelli and the participants of seminars at University of Wisconsin-Milwaukee and the 2008 XMU-HUB Workshop on Economics and Financial Econometrics for helpful comments and discussions. All the remaining errors are ours.
JEL Classification numbers: C14, C32.

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literature about estimation of QR models for time series. Engle and Manganelli (2004) propose a quantile autoregressive framework to model value-at-risk where the quantiles follow an autoregressive process. Gourieroux and Jasiak (2008) study dynamic additive quantile model. Xiao (2009) proposes QR with cointegrated time series. Recently, Xiao and Koenker (2009) studied conditional quantiles for GARCH models using QR.[1]

The purpose of this article is to generalize the Koenker and Xiao (2006) QAR framework introducing exogenous stationary covariates and to provide an application to illustrate the usefulness of the new model to study asymmetric behaviour in time series. We develop a quantile autoregressive distributed lag (QADL) model. The QADL model can deliver important insights about asymmetric dynamics, such as heterogeneous adjustments in time series models where controlling for lagged regressors and exogenous covariates is important. The approach proposed in this article is different from that of Engle and Manganelli (2004) because we use QR in the standard linear time series context, modelling the conditional quantile function as linear and depending on past values of the dependent variable, instead of modelling the quantile functions themselves as an autoregressive process. This reduces the computational burden substantially. Moreover, the QADL model allows for some forms of explosive behaviour in some quantiles while maintaining stationarity of the process, as long as certain stationarity conditions are satisfied on the whole distribution, while Engle and Manganelli (2004) exclude this case.[2]

Note that QAR and QADL in time series have a different interpretation than that of QR in cross-sectional data. In general, QR shows how a given quantile of the conditional distribution of y depends on the covariates x. In the cross-sectional case, this can be interpreted as the different effects that covariates exert on a given outcome for individuals on that corresponding quantile of the conditional distribution. In a time series context, however, we estimate the conditional quantile function of a particular variable along time, for instance aggregated variables such as GDP and consumption, index numbers, or as in the illustration presented in the article house price returns. Then, we interpret the conditional quantiles function at a given time as different phases of the business cycle, where low and high quantiles of the conditional distribution of price returns corresponds to periods of declining and increasing prices respectively. This interpretation might also be used for output gap, consumption growth or value-at-risk applications.

We illustrate the QADL model with an application to quarterly house price returns data in the United Kingdom (UK). House prices volatility has claimed unprecedented importance and there is a growing literature on this topic (for instance Muellbauer and Murphy, 1997; Ortalo-Magné and Rady, 1999, 2006; Rosenthal, 2006). We argue that QR can be used to describe the asymmetric responses of house prices returns to income and interest rates shocks. We interpret the conditional quantile functions as different phases of the market. High quantiles correspond to a phase of unusually high conditional returns; while

[1]Koenker and Xiao (2004) study statistical inference in QAR models when the largest autoregressive coefficient may be unity. Galvao (2009) develops tests for unit roots allowing for stationary covariates and a linear time trend into the quantile autoregression model.

[2]We do not consider the Xiao (2009) case where the variables are cointegrated, but rather we consider an exogenous set of stationary covariates.

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low quantiles to low conditional returns. The results show that house price returns have an asymmetric autoregressive behaviour, and that real GDP growth and interest rates have an asymmetric impact on house prices returns along the quantiles. In addition, the results suggest high autoregressive persistence in the extreme high quantiles. However, unit root tests reject the null hypothesis of unit root on house price returns. Thus, the model seems to show global stationarity with some persistence in unusually high returns. The inclusion of stationary covariates reduces the asymmetric autoregressive responses but maintains the persistence in the high quantiles. The interest rates have a negative impact on house prices returns, mostly significant for low quantiles. This can be interpreted as the fact that the interest rate has an effect on stimulating the demand in the real estate market when returns are low, but it does not deter house prices booms. In addition, there is evidence that the impact of GDP on house prices presents an asymmetric effect and it is stronger for low and high quantiles. For low quantiles, this is interpreted as the fact that GDP growth reactivates the real estate market when returns are low, while it might be contributing to house prices' busts (as that in the early 1990s where a recession was accompanied by a significant decline in house prices). Moreover, it contributes to sustaining house prices booms. In other words, periods of unusually high (conditional) returns are very responsive to GDP growth. Thus, the conditional mean may be a misleading parameter in periods of low and high conditional returns, which are those when policymakers are more keen to intervene or to predict future behaviour.

The rest of the article is organized as follows. Section II presents the model, describes the estimator and its asymptotic properties. Section III presents some Monte Carlo evidence. In section IV, we illustrate the new approach by applying it to a house price returns dataset. Finally, section V concludes the article.

II. QADL: model, estimation and inference

The autoregressive-distributed lag model is described by the following equation

$$y_t = \mu + \sum_{j=1}^{p} \alpha_j y_{t-j} + \sum_{l=0}^{q} x'_{t-l}\theta_l + \varepsilon_t; \quad t = 1, \ldots, n, \tag{1}$$

where y_t is the response variable, y_{t-j} is the lag of the response variable, x_t is a dim(x)-dimensional vector of covariates and ε_t is the innovation.[3] The main aim of this type of model is to emphasize alternative short-run dynamic structures. In addition, this class of models also provides important long-run results that are of particular interest for inference about the validity of a proposed economic theory. Nevertheless, the least squares models might be insufficient to describe heterogeneity in the impact of the shocks in a given time series.

As in Koenker and Xiao (2006), let $\{U_t\}$ be a sequence of independent and identically distributed (i.i.d.) standard uniform random variables, and consider the following autoregressive-distributed lag process

[3]We assume, for convenience, that each variable in x_t have the same lag truncation, q. The case of different lag truncation for each variable is immediate.

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$$y_t = \mu\left(U_t\right) + \sum_{j=1}^{p} \alpha_j\left(U_t\right) y_{t-j} + \sum_{l=0}^{q} x'_{t-l}\theta_l\left(U_t\right), \tag{2}$$

where α and θ are unknown functions $[0,1] \to \mathbb{R}$ that we want to estimate. Given that the right-hand side of equation (2) is monotone increasing on U_t, it follows that the τth conditional quantile function of y_t can be written as

$$Q_{y_t}\left(\tau \mid \Im_t\right) = \mu(\tau) + \sum_{j=1}^{p} \alpha_j(\tau) y_{t-j} + \sum_{l=0}^{q} x'_{t-q}\theta_l(\tau), \tag{3}$$

where $\Im_t$ is the σ-field generated by $\{y_s, x_s, s \leq t\}$.[4] We refer to model (3) as the quantile autoregressive distributed-lag of orders p and q (QADL(p, q)). Implicitly in the formulation of model (3) is the requirement that $Q_{y_t}(\tau \mid \Im_t)$ is monotone increasing in τ for all $\Im_t$. A more compact notation to describe model (3) is

$$Q_{y_t}\left(\tau \mid \Im_t\right) = z'_t\beta(\tau), \tag{4}$$

where $z_t = (1, y_{t-1}, \ldots, y_{t-p}, x_t, \ldots, x_{t-q})'$ and $\beta(\tau) = (\mu(\tau), \alpha_1(\tau), \ldots, \alpha_p(\tau), \theta'_0(\tau), \ldots, \theta'_q(\tau))'$.

It is important to emphasize that monotonicity of the conditional quantile functions imposes some discipline on the forms taken by the coefficients. It requires that the function $Q_{y_t}(\tau \mid \Im_t)$ is monotone in τ in a relevant region of the $\Im_t$-space. In some circumstances, this necessitates restricting the domain of the dependent variables; in others, when the coordinates of the dependent variables are themselves functionally dependent, monotonicity may hold globally. The estimated conditional quantile function $\hat{Q}_{y_t}(\tau \mid \Im_t) = z'_t\hat{\beta}(\tau)$ is ensured to be monotone in τ at $z_t = \bar{z}$, as noted in Koenker and Xiao (2006). However, this does not guarantee that it will be monotone in τ for other values of z. Furthermore, because we are using a linear model, there must be crossing sufficiently far away from $\bar{z}$. It may be that such crossing occurs outside the convex hull of the z observations, in which case the estimated model may be viewed as an adequate approximation within this region. But it is not unusual to find that the crossing has occurred in this region as well.[5] As discussed in Koenker and Xiao (2006), one can find a linear reparametrization of the model that does exhibit co-monotonicity over some relevant region of covariate space. Recently, Gourieroux and Jasiak (2008) propose a dynamic additive quantile model that ensures the monotonicity of conditional quantile estimates.

The estimation procedure is based on standard linear quantile regression. Thus, estimation of the QADL model (3) involves solving the following problem

[4]The transition from equation (2) to equation (3) is an immediate consequence of the fact that for any monotone increasing function g and standard uniform random variable, U, we have $Q_{g(U)}(\tau) = g(Q_U(\tau)) = g(\tau)$, where $Q_U(\tau) = \tau$ is the quantile function of U.

[5]It is easy to check whether $\hat{Q}_{y_t}(\tau \mid \Im_t)$ is monotone at particular z points. To verify monotonicity for a given z, one may compute this for several quantiles, and plot it against the sequence of τ. If there is a significant number of observed points at which this condition is violated, then this can be taken as evidence of model misspecification. Failure of the monotonicity condition might also imply that the conditional quantile functions are not linear. In this article, we assume that monotonicity of $Q_{y_t}(\tau \mid \Im_t)$ in τ, for some relevant region of $\Im_t$-space, holds. We refer the reader to Gourieroux and Jasiak (2008), Neocleous and Portnoy (2008), Koenker and Xiao (2006), and Koenker (2005) for more details about monotonicity in QR.

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$$\min_{\beta \in \Re^{1+p+(1+q)\times \dim(x)}} \sum_{t=1}^{n} \rho_\tau \left(y_t - z_t'\beta \right) \tag{5}$$

where $\rho_\tau(u) = u(\tau - I(u < 0))$, as in Koenker and Bassett (1978). The details of the proofs for consistency and asymptotic normality of the estimator, $\hat{\beta}(\tau)$, are provided in Galvao, Montes-Rojas and Park (2009). Define the following elements: $\Omega_0 = E(z_t z_t') = \lim n^{-1} \sum_{t=1}^{n} z_t z_t'$, and $\Omega_1(\tau) = \lim n^{-1} \sum_{t=1}^{n} f_{t-1}[F_{t-1}^{-1}(\tau)] z_t z_t'$, and let $\Sigma(\tau) = \Omega_1(\tau)^{-1} \Omega_0 \Omega_1(\tau)^{-1}$. The limiting distribution of the QADL estimator for a fixed quantile τ is

$$\sqrt{n} \left(\hat{\beta}(\tau) - \beta(\tau) \right) \xrightarrow{d} N(0, \tau(1-\tau)\Sigma(\tau)).$$

In order to make appropriate inference, it is necessary to estimate $\Sigma(\tau)$ consistently. Since Ω_0 involves no nuisance parameter, it can easily be estimated as $\hat{\Omega}_0(\tau) = \frac{1}{n} \sum_{t=1}^{n} z_t z_t'$. Let $\hat{u}_t(\tau) = y_t - z_t'\hat{\beta}(\tau)$, in order to estimate the matrix $\Omega_1(\tau)$, we follow Powell (1986)

$$\hat{\Omega}_1(\tau) = \frac{1}{2nh_n} \sum_{t=1}^{n} I(|\hat{u}_t(\tau)| \le h_n) z_t z_t',$$

where h_n is an appropriately chosen bandwidth, with $h_n \to 0$ and $nh_n^2 \to \infty$.[6]

In model (3) the choice of p and q is important. In order to select appropriate models, we suggest the use of BIC criteria, adapted to QADL along the lines suggested by Machado (1993), which is based on the Asymmetric Laplace Distribution. At the median, it uses the criterion

$$\mathrm{BIC} = n \log \hat{\sigma} + \frac{1 + p + (1+q) \times \dim(x)}{2} \log n,$$

where $\hat{\sigma} = n^{-1} \sum |y_t - z_t'\hat{\beta}(1/2)|$. For other quantiles, the obvious asymmetric modification of this expression can be used. In the example given in this article, we select the number of lags based only on the median criterion, in order to have a comparable regression model across quantiles. But, it is possible that there are applications in which this is not desirable.

General hypotheses tests on the vector $\beta(\tau)$ can be accommodated by Wald-type tests (see Galvao *et al.*, 2009). The Wald process and associated limiting theory provide a natural foundation for the hypothesis $R\beta(\tau) = r$, when r is known. Here R is a $k \times (1 + p + (1+q)\dim(x))$ matrix with rank k and r is a k-dimensional vector. This formulation also accommodates a wide variety of testing situations, from a simple test on single QR coefficients to joint tests involving several parameters and distinct quantiles. Thus, for instance, we might test for the equality of several slope coefficients across several quantiles. Another important class of tests in the QR literature involves the Kolmogorov–Smirnov (KS) type tests, where the interest is to examine the property of the estimator over a range of quantiles $\tau \in \mathcal{T}$, instead of focusing only on selected quantiles. Thus, for testing $R\beta(\tau) = r$ over $\tau \in \mathcal{T}$, one may consider the KS type sup-Wald test.

[6]In the simulations and application, we consider the default bandwidth suggested by Bofinger (1975), $h_n = [\Phi^{-1}(\tau + c_n) - \Phi^{-1}(\tau - c_n)] \min(\hat{\sigma}_1, \hat{\sigma}_2)$, where the bandwidth $c_n = O(n^{1/3})$, $\hat{\sigma}_1 = \sqrt{\mathrm{Var}(\hat{u})}$, and $\hat{\sigma}_2 = (\hat{Q}(\hat{u}, .75) - \hat{Q}(\hat{u}, .25))/1.34$.

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III. Monte Carlo

In this section, we briefly present simulation experiments to assess the finite sample performance of the QADL estimator. Two simple versions of the basic model (3) are considered in the experiments. In the first version, reported in Table 1, the scalar covariate, x_t, exerts a pure location-shift effect, and the response y_t is generated by the model

$$y_t = \alpha y_{t-1} + \beta_1 x_t + \beta_2 x_{t-1} + u_t. \tag{6}$$

In the second, reported in Table 2, x_t exerts both location and scale effects as

$$y_t = \alpha y_{t-1} + \beta_1 x_t + \beta_2 x_{t-1} + (\gamma x_t) u_t. \tag{7}$$

We employ two different distributions to generate the disturbances u_t: $N(0, 1)$ and t-distribution with 3 degrees of freedom (t_3). In all cases, we set $y_0 = 0$ and generate y_t for $t = 1, \dots, n$ according to equations (6) and (7), and in generating y_t we discarded the first 100 observations, using the remaining observations for estimation. This ensures that the results are not unduly influenced by the initial values of the y_0 process. In the location case, we generate the exogenous covariates, x_t, using the same distribution as the innovations u_t. In the location-scale version, to avoid crossing, we generate covariates x_t as χ^2_3. In the simulations, we use a sample size of $n = 200$, set the number of replications to 5,000, and consider the following values for the remaining parameters: $(\alpha, \beta_1, \beta_2) = (0.5, 0.5, 0.5)$ and $\gamma = 0.2$. We compare the estimators' coefficients in terms of bias and root mean squared error (RMSE).[7] We study four different estimators in the Monte Carlo experiments, the QAR proposed by Koenker and Xiao (2006), the QADL proposed in this article, the least squares estimator (OLS), and finally, the least square distributed lag model (ADL). Finally, we consider $\tau \in \{0.1, 0.25, 0.5, 0.75, 0.9\}$.

Table 1 shows bias and RMSE results of the estimators for the location-shift model. For QAR and OLS models, we do not include the terms x_t and x_{t-1} in the estimating equation. The results show that, as expected, omitting the variables in QAR and OLS cause bias in estimation. However, QADL and ADL are approximately unbiased, with QADL unbiased along the quantiles. The results show that for the quantile regression models the RMSE is larger for extreme quantiles in both distributions. This finding shows evidence that, in general, it is more difficult to estimate the variance at the extremes of the distribution. We compare RMSE of the least squares estimators with the median quantile regression. In the Gaussian condition, the OLS based estimators outperform the quantile regression estimators, that is, ADL has smaller RMSEs when compared with QADL, and OLS has smaller RMSE when compared with QAR. Finally, for the non-Gaussian case, t_3, in terms of RMSE, the median quantile regression estimators outperform their least squares analogues. Table 2 shows bias and RMSE results of the estimates of α and β for location-scale-shift model. In all cases, the QAR and OLS are biased and the QADL and ADL are approximately unbiased. In this case, the results regarding RMSE are qualitatively similar to the previous case.

[7]We refer the reader to Galvao *et al.* (2009) for a more detailed set of results on estimation, and small sample properties (size and power) of the Wald and Kolmogorov–Smirnov tests.

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TABLE 1

Location-shift model: bias and root mean squared error (RMSE) of estimators

		Quantile autoregression (QAR)					*Quantile autoregressive distributed lag (QADL)*					*Ordinary least square (OLS)*	*Autoregression distributed lag (ADL)*
	τ	0.1	0.25	0.5	0.75	0.9	0.1	0.25	0.5	0.75	0.9	–	–
N	α	0.094	0.093	0.095	0.095	0.099	−0.006	−0.007	−0.008	−0.005	−0.006	0.095	−0.007
		(0.136)	(0.126)	(0.130)	(0.128)	(0.138)	(0.087)	(0.076)	(0.077)	(0.076)	(0.086)	(0.111)	(0.050)
	β_1						0.000	−0.001	0.000	0.007	0.000		0.001
							(0.122)	(0.109)	(0.111)	(0.108)	(0.123)		(0.072)
	β_2						0.006	0.006	0.006	0.000	0.010		0.005
							(0.131)	(0.115)	(0.117)	(0.114)	(0.129)		(0.076)
t_3	α	0.110	0.086	0.090	0.082	0.105	−0.007	−0.006	−0.005	−0.006	−0.009	0.098	−0.007
		(0.150)	(0.113)	(0.109)	(0.111)	(0.145)	(0.087)	(0.046)	(0.045)	(0.049)	(0.086)	(0.113)	(0.050)
	β_1						0.000	0.001	0.001	0.001	−0.002		0.001
							(0.131)	(0.078)	(0.071)	(0.080)	(0.131)		(0.118)
	β_2						0.008	0.003	0.002	0.002	0.003		0.002
							(0.136)	(0.078)	(0.075)	(0.078)	(0.141)		(0.128)

Notes: RMSE in parenthesis. Sample size of $n=200$. The number of replications is 5,000.

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TABLE 2

Location-scale-shift model: bias and root mean squared error (RMSE) of estimators

		Quantile autoregression (QAR)					*Quantile autoregressive distributed lag (QADL)*					*Ordinary least square (OLS)*	*Autoregression distributed lag (ADL)*
	τ	0.1	0.25	0.5	0.75	0.9	0.1	0.25	0.5	0.75	0.9	–	–
N	α	0.090	0.113	0.143	0.173	0.190	−0.005	−0.005	−0.003	−0.005	−0.008	0.144	−0.005
		(0.118)	(0.136)	(0.165)	(0.192)	(0.222)	(0.066)	(0.059)	(0.060)	(0.059)	(0.067)	(0.154)	(0.042)
	β_1						0.009	0.004	0.002	−0.002	−0.012		0.002
							(0.112)	(0.094)	(0.093)	(0.094)	(0.111)		(0.049)
	β_2						0.005	0.002	0.002	0.003	0.000		0.002
							(0.085)	(0.075)	(0.077)	(0.075)	(0.085)		(0.053)
t_3	α	0.066	0.075	0.084	0.100	0.105	−0.008	−0.006	−0.004	−0.005	−0.006	0.092	−0.008
		(0.102)	(0.096)	(0.110)	(0.125)	(0.153)	(0.070)	(0.050)	(0.047)	(0.045)	(0.079)	(0.105)	(0.062)
	β_1						0.011	0.003	0.003	−0.005	−0.004		−0.002
							(0.185)	(0.115)	(0.106)	(0.114)	(0.187)		(0.085)
	β_2						−0.005	0.001	0.001	0.005	0.012		0.006
							(0.137)	(0.088)	(0.086)	(0.088)	(0.136)		(0.089)

Notes: RMSE in parenthesis. Sample size of $n = 200$. The number of replications is 5,000.

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IV. Application: house price returns

There is an extensive literature on cross-sectional and time-series variation in house prices, but this literature is marked by poor predictability. Mankiew and Weil (1989) find that the Baby Boom had a large impact on the US housing market. By 1989, they predicted a future slow down in the house market, which was not observed.[8] In fact, house prices have shown unprecedented values over the past 10 years. Increasing house prices had also importance in the UK. The issue of affordable housing had claimed an increasing importance in the public debate and the uncertainty about future prices is a concern of both policy makers and researchers.[9] Moreover, the fact that housing is a major component of wealth (Banks and Tanner, 2002, show that real state accounted for 35% of aggregate household wealth in the UK in the 1990s) and risky assets determines that house price changes have significant effects on aggregate consumption (see for instance Campbell and Cocco, 2007).

The evolution of house prices was extensively studied in the UK by Muellbauer and Murphy (1997), Ortalo-Magné and Rady (1999, 2006) and Rosenthal (2006) among others. Those authors rigorously studied the booms and busts in the UK housing market until 2000. In the past 50 years, there have been three major booms in the UK's owner-occupied housing market: in the early 1970s, in the late 1980s and the current housing boom. There were also smaller booms in the 1960s and, more briefly, in the late 1970s, while the early 1990s saw a bust on an unprecedented scale. Many factors conspired to produce the house price boom of the late 1980s. Initial debt levels were low as were real house prices, giving scope for rises in both. Income growth after the early 1980s recession was strong, as were income growth expectations and these became more important as a result of financial liberalization, though partly offset by bigger real interest rate effects. Wealth to income ratios grew and illiquid assets increased enhanced by financial liberalization. Financial liberalization also permitted higher gearing levels. Demographic trends were favourable with stronger population growth in the key house buying age group. The supply of houses grew more slowly, with construction of social housing falling to a small fraction of its level in the 1970s. Finally, in 1987–88 interest rates fell and the proposed abolition of property taxes in favour of the Poll Tax gave a further impetus to valuations.

The bust in the early 1990s was the result of the reversal of most of these factors. Interest rates rose from 1988 to 1990. The bust coincided with a general recession. Demographic trends reversed. The revolt against the Poll Tax resulted in a new property tax, the Council Tax, being reintroduced. Debt levels and real house prices had reached very high levels, while wealth to income ratios then fell and recently experienced rates of return became negative and made households more cautious. Mortgage lenders tightened up their lending criteria, in a partial reversal of financial liberalization. Under these conditions, not even the major falls in nominal interest rates that took place in the early 1990s, while real interest rates remained high, were sufficient to revive UK house prices. However, the late 1990s and the new millennium showed an unprecedented increase in house prices, mostly concentrated in the Southeast (i.e. London).

[8]'Our estimates suggest that real housing prices will fall substantially – indeed, real housing prices may well reach levels lower than those experienced at any time in the past forty years' (p. 236).

[9]'Our forecasts have not been for dramatic falls in house prices, but who knows?... anyone who thinks that they can forecast asset prices is kidding themselves'. Rachel Lomax, Governor of Bank of England, The Guardian, London, November 23, 2003, cited in Rosenthal (2006, p. 289).

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The conditional quantiles provide a complete picture of the distribution of house returns conditional on past values. High quantiles correspond to unusually high conditional returns; low quantiles correspond to busts in the conditional returns. We propose the application of QADL to model house price returns in order to study the asymmetric behaviour of this time series. We are particularly interested in the autoregressive behaviour of this series at different quantiles, as well as the response to income shocks and the interest rate.

House price series are obtained from Nationwide mortgage data. Nationwide Building Society has a long history of recording and analyzing house price data and has published average house price information since 1952 while the quarterly data used here started in 1973. It is the 4th largest mortgage lender in the UK by stock. The series used in this application is the average price of a representative house, UK Quarterly Index. This series is constructed by Nationwide using mortgages that are at the approvals stage and after the corresponding building survey has been completed. Approvals data is used as opposed to mortgage completions since it should give an earlier indication of current trends in prices in the residential housing market. In addition, properties that are not typical and may distort the series are also removed from the data set. The index controls for: location in the UK, type of neighbourhood, floor size, property design (detached house, semi-detached house, terraced house, bungalow, flat, etc.), tenure (freehold/leasehold/feudal, except for flats, which are nearly all leasehold), number of bathrooms (1 or more than 1), type of central heating (full, part or none), type of garage (single garage, double garage or none), number of bedrooms (1,2,3,4 or more than 4), and whether property is new or not.

The series in levels are shown in Figure 1. Nominal prices provide a quick overview of the magnitude of the increase in house prices. With an average value of £25,000 in 1975, the latest estimate is close to £200,000. Even when adjusting by inflation, the recent increments are significant. The current boom in house prices can be seen by the continuous growth in the past 12 years. Overall, all series show a similar performance in terms of business cycle patterns. The series show three different cycles over the past 35 years with spikes in 1980, 1990 and possibly in 2007. Interest rates are currently at a record low. Real house price returns and real GDP growth series are shown in Figure 2.

In the long run, we expect that house price variations depend on its past values and some key economic variables. Based on Muellbauer and Murphy (1997), we propose an autoregressive specification of quarterly house price returns in UK using the quantile autoregressive distributed lag model. As additional covariates we use the Bank of England interest rates, real GDP growth and dummy variables for quarter effects. The proposed model is given by

$$Q_{r_t}\left(\tau \mid \Im_t\right) = \mu(\tau) + \sum_{j=1}^{p} \alpha_j(\tau) r_{t-j} + \sum_{k=0}^{q_1} \gamma_k(\tau) g_{t-k} + \sum_{l=1}^{q_2} \theta_l(\tau) i_{t-l}$$
$$+ \beta_1(\tau) D_{1,t} + \beta_2(\tau) D_{2,t} + \beta_3(\tau) D_{3,t},$$

where r_t is the real quarterly price return in period t, obtained as the difference in the natural logarithm of house prices (deflated by the consumer price index), g_t is the growth rate of real GDP, i_t is the interest rates, and D represent dummy variables for quarter effects. Note that when we exclude the covariates and the quarter dummy variables, we have the QAR model. Augmented Dickey–Fuller (ADF) tests are applied to these variables to check

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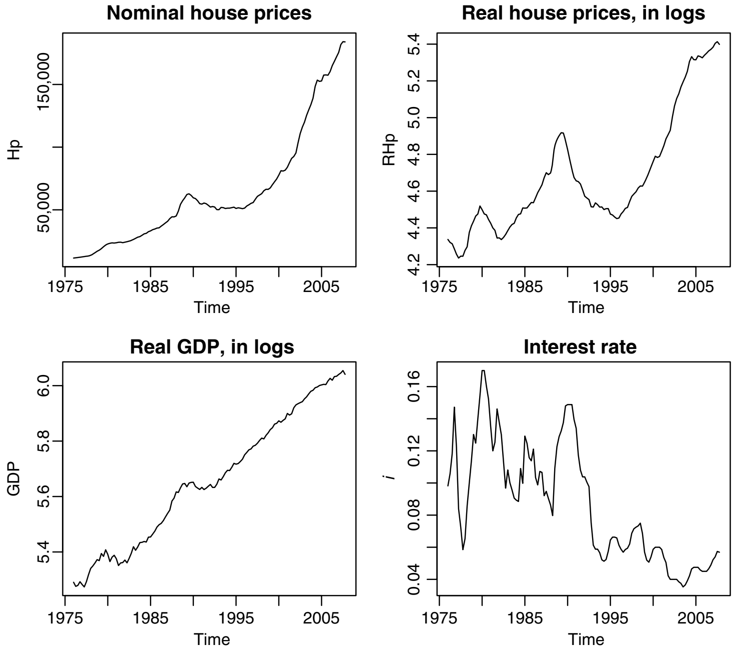

Figure 1. Time series

for unit roots. The r_t series has an ADF value of -3.91 with a corresponding p-value for the null hypothesis of unit root of 0.012. i_t has an ADF value of -3.70 with a p-value of 0.026. g_t has an ADF value of -3.89 with a p-value of 0.017. Therefore, for all the variables we reject the unit root null hypothesis.

We first estimate the QAR model using Koenker and Xiao (2006) methodology. We use the BIC criteria as developed in Machado (1993) for $\tau = 0.5$ to determine the number of lags, and this suggests using a QAR model with $p = 1$. This is in line with Rosenthal (2006) findings that suggest that 2- to 3-month lags are enough to model monthly house prices. Although not reported, we also apply the BIC criteria for a range of $\tau \in [0.05, 0.95]$, and in general, the model with one lag is selected, which determines that the selection for the median may be appropriate for the whole distribution. Next, we perform the QAR estimation for several quantiles and the results appear in Figure 3 (QAR(1) alpha) that plots the coefficient estimates with 95% confidence interval. The results show a strong asymmetry in the lag response. Unit-root like behaviour is observed for high quantiles. However, when we perform Koenker and Xiao (2004) QR unit-root tests for all $\tau \in [0.05, 0.95]$, we always reject the null hypothesis of unit-roots. Then, overall, the QAR(1) process is globally stationary.

Next, we consider our suggested QADL model. Applying a similar BIC criteria we select $p = 1$, $q_1 = 0$ for GDP, i.e. the contemporaneous effect of GDP, and $q_2 = 0$ for the interest rate (although we exclude the contemporaneous effect, that is, we consider the interest rate lagged one quarter only). The estimates shown in Figure 3 (QADL(1,0) alpha) suggests that the model still shows unit-root-like behaviour only in the high extreme quantiles. However, when we perform Galvao (2009) QR unit-root tests for all $\tau \in [0.05, 0.95]$ we always reject the null hypothesis of unit-roots. That is, the model seems to show global

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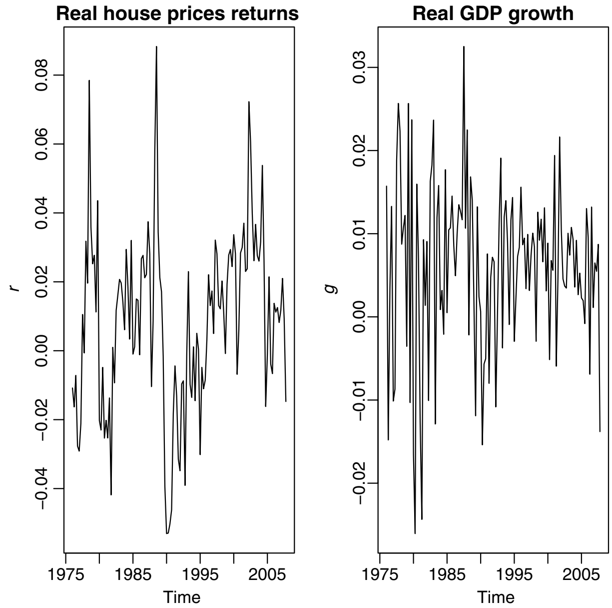

Figure 2. Time series

stationarity with higher persistence in unusually high shocks. Note that the inclusion of the covariates determines a more homogeneous increasing behaviour of the α-coefficients along different quantiles than that observed in the QAR model.

The interest rate has a negative impact on house price returns, although only statistically significant for low quantiles (see Figure 3, QADL(1,0) theta). In other words, this variable may have an effect to prevent busts, but it may not deter house price booms. Therefore, the policy followed by the Bank of England of cutting the interest rate to prevent a house price collapse may have the desired effect. A Kolmogorov–Smirnov test of the hypothesis that $\sup_{\tau\in\mathcal{T}}\theta_1(\tau)=0$ gives a KS value of 12.8. Looking at Andrews (1993, p. 840) the critical values are 8.19, 9.84, 13.01 for 10%, 5% and 1% significance levels respectively. Therefore, the interest rate has an effect different from zero at the 5% significance level.

Real GDP growth has a larger impact on low and high quantiles than for medium quantiles (see Figure 3, QADL(1,0) gamma). For low quantiles, this is interpreted as the fact that GDP growth reactivates the housing market when returns are low, while it might be contributing to house prices' busts (as that in the early 1990s). Moreover, it contributes to sustaining house prices increments. In other words, periods of unusually conditional high returns are very responsive to GDP growth. Note that the estimated coefficient for very high quantiles is greater than 1, although not statistically different from this value except for a few quantiles. Poterba (1991) and Capozza *et al.* (2002) among others, provide evidence on the asymmetric responses of house prices to income shocks. The QADL estimates present this effect of house prices to income shocks, but restricted to high quantiles. A Kolmogorov–Smirnov test of the hypothesis that $\sup_{\tau\in\mathcal{T}}\gamma_0(\tau)=0$ gives a KS value of 33.1, which by the critical values discussed above show that the effect of GDP is not zero (as expected from the figure). However, the hypothesis that $\sup_{\tau\in\mathcal{T}}\gamma_0(\tau)=1$ gives KS = 4.1. Then, overall, the effect of GDP growth on house price returns is not different from 1.

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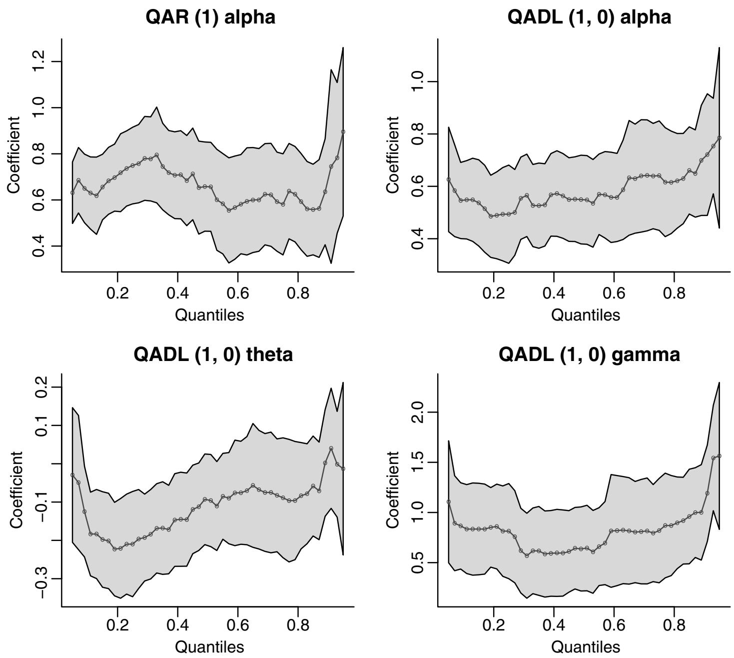

Figure 3. Coefficient estimates

In summary, the application illustrates the usefulness of the QADL process to model asymmetric behaviour in time series. Of particular importance are the asymmetries in the slope of the lagged dependent variable and other covariates in both extreme low and high quantiles. In this case, the conditional mean may be a misleading parameter in periods of extremely low and high conditional returns, which are those when policymakers are keener to intervene or to predict future behaviour.

V. Conclusion

We have developed a quantile autoregression distributed lag model (QADL). Quantile regression methods provide a framework for robust estimation and inference and allow one to explore a variety of forms of conditional heterogeneity under less compelling distributional assumptions. The proposed model is able to accommodate exogenous covariates in the QAR model. Monte Carlo simulations are conducted to evaluate the finite sample performance of the QADL estimator. It is shown that the simple quantile autoregression estimator is severely biased by omitting exogenous variables, while the QADL is generally unbiased. In addition, the QADL approach outperforms the ordinary augmented distributed lag approach in terms of root mean square error for non-Gaussian heavy tail distributions.

We illustrate the QADL model with an application to quarterly house price returns data in the UK. The results show that house price returns have an asymmetric autoregressive behaviour, and that real GDP growth and interest rates have an asymmetric impact on house prices variations along the quantiles. In addition, the results suggest that unit root behaviour is present only in the high extreme quantiles. Thus, the model seems to show global stationarity with some persistence in unusually high returns. The inclusion

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of covariates determines a more homogeneous increasing behaviour of the autoregressive coefficients along different quantiles than that observed in the QAR model, but maintains the persistence in the high quantiles. The interest rate has a negative impact on house prices, mostly significant for low quantiles. This can be interpreted as the fact that interest rates have an effect on stimulating the demand in the real estate market when returns are low, but it does not deter house prices booms. In addition, there is evidence that the impact of GDP on house prices presents an asymmetric persistence and it is stronger for low and high quantiles. For low quantiles, this is interpreted as the fact that GDP growth reactivates the real estate market when returns are low, while it might be contributing to house prices' busts. Moreover, it contributes to sustaining house prices booms. In other words, periods of unusually high returns are very responsive to GDP growth.

Final Manuscript Received: October 2011

References

Andrews, D. W. K. (1993). 'Tests for parameter instability and structural change with unknown change point', *Econometrica*, Vol. 61, pp. 821–856.

Banks, J. and Tanner, S. (2002). 'Household portfolios in the United Kingdom', in Guiso L., Haliassos M. and Japelli, T. (eds), *Household Portfolios*, Chapter 6, pp. 219–250, MIT Press, Cambridge.

Beaudry, P. and Koop, G. (1993). 'Do recessions permanently change output?' *Journal of Monetary Economics*, Vol. 31, pp. 149–163.

Bofinger, E. (1975). 'Estimation of a density function using order statistics', *Australian Journal of Statistics*, Vol. 17, pp. 1–7.

Campbell, J. and Cocco, J. (2007). 'How do house prices affect consumption? evidence from micro data', *Journal of Monetary Economics*, Vol. 54, pp. 591–621.

Capozza, D. R., Hendershott, P. H., Mack, C. and Mayer, C. J. (2002). *Determinants of Real House Price Dynamics*, NBER Working Paper Series 9262.

Engle, R. F. and Manganelli, S. (2004). 'Caviar: conditional autoregressive value at risk by regression quantiles', *Journal of Business and Economic Statistics*, Vol. 22, pp. 367–381.

Galvao, A. F. (2009). 'Unit root quantile autoregression testing using covariates', *Journal of Econometrics*, Vol. 152, pp. 165–178.

Galvao, A. F., Montes-Rojas, G. and Park, S. Y. (2009). *Quantile Autoregressive Distributed Lag Model with an Application to House Price Returns*, City University London, Department of Economics Discussion Paper Series 09/04.

Gourieroux, C. and Jasiak, J. (2008). 'Dynamic quantile models', *Journal of Econometrics*, Vol. 147, pp. 198–205.

Koenker, R. (2005). *Quantile Regression*, Cambridge University Press, New York.

Koenker, R. and Bassett, G. W. (1978). 'Regression quantiles', *Econometrica*, Vol. 46, pp. 33–49.

Koenker, R. and Xiao, Z. (2004). 'Unit root quantile autoregression inference', *Journal of the American Statistical Association*, Vol. 99, pp. 775–787.

Koenker, R. and Xiao, Z. (2006). 'Quantile autoregression', *Journal of the American Statistical Association*, Vol. 101, pp. 980–990.

Machado, J. A. F. (1993). 'Robust model selection and m-estimation', *Econometric Theory*, Vol. 9, pp. 478–493.

Mankiew, N. G. and Weil, D. N. (1989). 'The baby boom the baby bust and the housing market', *Regional Science and Urban Economics*, Vol. 19, pp. 235–258.

Muellbauer, J. and Murphy, A. (1997). 'Booms and busts in the UK housing market', *The Economic Journal*, Vol. 107, pp. 1701–1727.

Neocleous, T. and Portnoy, S. (2008). 'On monotonicity of regression quantile functions', *Statistics and Probability Letters*, Vol. 78, pp. 1226–1229.

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Ortalo-Magné, F. and Rady, S. (1999). 'Boom in, bust out: young households and the housing price cycle', *European Economic Review*, Vol. 43, pp. 755–766.

Ortalo-Magné, F. and Rady, S. (2006). 'Housing market dynamics: on the contribution of income shocks and credit constraints', *Review of Economic Studies*, Vol. 73, pp. 459–485.

Poterba, J. (1991). 'House price dynamics: the role of tax policy and demography', *Brookings Papers on Economic Activity*, Vol. 2, pp. 143–203.

Powell, J. L. (1986). 'Censored regression quantiles', *Journal of Econometrics*, Vol. 32, pp. 143–155.

Rosenthal, L. (2006). 'Efficiency and seasonality in the UK housing market, 1991–2001', *Oxford Bulletin of Economics and Statistics*, Vol. 68, pp. 289–317.

Xiao, Z. (2009). 'Quantile cointegrating regression', *Journal of Econometerics*, Vol. 150, pp. 248–260.

Xiao, Z. and Koenker, R. (2009). 'Conditional quantile estimation for generalized autoregressive conditional heteroscedasticity models', *Journal of the American Statistical Association*, Vol. 104, pp. 1696–1712.

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