XI

Volume XI, Number 1, May 2008

Journal of Applied Economics

Gabriel Montes-Rojas

Non-uniform wealth distribution in a simple spatial banking model

UCEMA

Edited by the Universidad del CEMA

Print ISSN 1514-0326
Online ISSN 1667-6726

Journal of Applied Economics. Vol XI, No. 1 (May 2008), 145-165

NON-UNIFORM WEALTH DISTRIBUTION IN A SIMPLE SPATIAL BANKING MODEL

Gabriel Montes-Rojas*
University of Illinois at Urbana-Champaign
and
City University London

Submitted December 2005; accepted January 2007

This paper uses a static spatial banking model with a non-uniform wealth distribution to provide theoretical assessments for differences in banks' prices and locations across regions. It assumes imperfect information, where banks know more about individuals if they are "near" the bank and individuals incur in a cost proportional to this distance to show the viability of their projects to the bank. A free entry model is constructed to account for banks' tendency to concentrate in rich regions and to charge lower prices. Comparative statics exercises show the effect of changes in the monitoring technology and wealth dispersion.

JEL classification codes: G21, L11

Key words: spatial banking, location

I. Introduction

The non-competitive behavior of the banking industry has been extensively documented. The available evidence on the banking industry points to the fact that, despite technological advances that reduced the cost of information flows, regional aspects of the industry play an important role. For instance, Petersen and Rajan (2002) show that the distance to the nearest bank is a good predictor of the cost that small firms face for obtaining credit. Moreover, Hannan (1991) finds that the commercial loan market in the United States is local instead of national. Keeton (1998) shows that the cost of receiving a loan from a bank outside the rural area is higher than receiving it from a local bank.

* Email: Gabriel.Montes-Rojas.1@city.ac.uk, tel: +44 (0)20 7040-8919, City University London, Department of Economics, Northampton Square, London, EC1V 0HB, United Kingdom. I am grateful to Jorge Streb and three anonymous referees for their comments and suggestions. I am also grateful to Osvaldo Schenone and Enrique Kawamura for their comments in the first version of this paper. All errors are mine.

Concentration and market size explain much of the inter-regional differences in prices of banking services. For instance, Pilloff and Rhoades (2002) provide empirical evidence on the effect of the market size and concentration measures on banks' profitability. They argue that larger market sizes are associated with lower profit rates, because higher profits are a magnet for entry and thus are unsustainable. They also show that income per capita (or wealth) is a possible indicator of market's attractiveness. In addition, wealthier customers may be able to shop for better banks and may hold large balances or utilize more profitable services. Berger and Hannan (1989) explain the relationship between concentration and profitability in terms of the non-competitive price behavior of banks and find a price-concentration relationship consistent with this hypothesis.

In developing countries, regional aspects of the banking market might be more important provided that worse information technologies are used. For Argentina, Burdisso and D'Amato (1999) show that the degree of concentration (as measured by the market share) is very small in the most populated areas of the country, while the opposite occurs in less developed areas. An immediate consequence is that banks contribute to enlarge regional disparities.

These facts motivate the use of a spatial model where geographic distance plays an important role. Several studies use spatial models to account for banking competition. For instance, Wong and Chan (1993) develop a standard Hotelling (1929) model and Chiappori, Perez-Castillo and Verdier (1995) use a Salop (1979) style model. They argue that those models are adequate because of the existence of "monitoring costs" that are proportional to the "distance" mentioned earlier. The space in which those distances develop can be regional, socio-cultural, etc. Dell'Ariccia (2001) interprets it as a general space of products where borrowers have preferences for a particular type of loan and uses a spatial model to formalize market power and strategic interaction.

This paper uses a static Hotelling-linear-city-style model with free entry and proportional monitoring costs. In this model, banks have some local market power over their clients, which is a consequence of the existence of those costs. The contribution of the paper is to introduce an unequal distribution of wealth. This will produce important results. First, regions with a high density of wealth will have low prices and more banks. This adds a potential explanation to the priceconcentration relationship in banking. Second, the average monitoring cost will be higher for low wealth density regions. A limitation of the model is that only one side of banks' activity, the lending process, is studied.

Finally the potential effect of the introduction of new technologies and changes in the distribution of wealth (as measured by a dispersion parameter) is explored. The former was extensively studied (see for instance Rajan 1992, Wilhem 1999, Marquez 2002, and Hauswald and Marquez 2004). The spatial model provides a simple framework to study the effect of the introduction of a new technology: lower monitoring costs produce higher dispersion in the location of banks but lower prices. The effect of changes in the concentration of wealth was studied only in two-firm models (see for instance Neven 1986, Tabuchi and Thisse 1995 and Anderson, Goeree and Ramer 1997). The model developed in the paper allows to study the overall distribution of many banks.

II. The model

Consider a modification of the Linear City model (Hotelling 1929). A country is represented by the real line and has inhabitants (or individuals) distributed uniformly (the measure of inhabitants is infinite). Let the continuous variable $x \in \Re$ represent the location of those individuals. Each inhabitant has an associated wealth represented by the measurable function $x \to f(x)$. If the distribution of wealth is uniform, $f(x)$ will be a constant (without loss of generality 1). If that is not the case, assume that

Assumption 1 (Non-uniform distribution of wealth): : $f\ \Re \to \Re$ is a measurable function which is twice continuously differentiable, and satisfies

(i) $f(x)$ has a unique maximum at $x=0$;

(ii) symmetry: $f(x)=f(-x)$;

(iii) $f(.) \geq 1$, $f'(x) \geq 0$ for $x < 0$, $f'(x) \leq 0$ for $x > 0$, $f'(0) = 0$;

(iv) $\int (f(x) - 1)\, dx = 1$

(v) $\lim_{|x| \to \infty} f(x) = 1, \lim_{|x| \to \infty} f'(x) = 0$

Moreover, as $|x| \to \infty$ wealth becomes uniformly distributed. Additionally, $x = 0$ represents the richest inhabitant of the country, and wealth decreases as $|x|$ becomes larger. A simple way to generate this wealth measure is to assume a uniform measure and to add a density function (e.g., a standard normal). Thus, as $|x|$ increases, the contribution of the density becomes negligible.

This standard set-up can be reinterpreted to accommodate a non-uniform distribution of inhabitants with the same wealth. In this case, $f(.)$ would become a population measure and therefore, high values of $f(.)$ would be associated with highly populated regions of the country. In any case, a region is defined as an interval on the real line. Therefore, a rich (poor) region is one with relative high (low) values of $f(.)$.

The country's inhabitants can consume their wealth or invest in projects that provide $v > 1$ goods per good invested (since our model is static we do not have a time discount factor). v is assumed to be large enough to assure competition in the model presented below. Thus, individuals with a wealth of $f(x)$ prefer to carry out the projects and get $v \times f(x)$. However, the projects need banks' services. Additionally, those projects are not directly observable by banks, unless the individuals pay a fee, which is proportional to the distance to the nearest bank. Call this cost h. Following Wong and Chan (1993) this cost will be referred vaguely as monitoring cost. Those authors assume that the bank pays the full cost of monitoring, but the assumption that the borrowers pay the fee h is consistent with the idea that they have to move to the bank's location and show their projects' viability. In a broader interpretation, banks may be better informed about the profitability of local producers' projects than about remote regions. Similarly information about local tax exemptions or subsidies may be available only for local projects.

Consider two infinite sequences of banks, indexed by $i = 0,1,2,\ldots$ and $k = 0,1,2,\ldots$, that play a three stage non-cooperative game. In the first stage $i = 0,1,2,\ldots$ simultaneously decide whether or not to enter the market (let $\{E_i\}_{i=0}^{\infty}$ be the entry decisions) , and if they do, they choose a location $\{z_i\}_{i=0}^{\infty}$ where zi denotes location on the real line (two banks cannot locate at the same point).[1] Banks $k = 0,1,2,\ldots$ may enter the market immediately after the first stage is decided (second stage). Denote by $\{E_k\}_{k=0}^{\infty}$ the entry decisions and by $\{y_k\}_{k=0}^{\infty}$ the locations of the second-stage banks.

In the third stage they compete in prices. Banks can charge two different prices (one to the left and one to the right), denoted by $\{P_i^-, P_i^+\}_{i=0}^{\infty}$ and $\{P_k^-, P_k^+\}_{k=0}^{\infty}$ for the first-stage and second-stage banks respectively. This price discrimination will simplify the computation of the equilibrium without altering the qualitative conclusions. Each bank can enter only through one branch and face an entry cost given by $0 < s < \infty$ and a constant cost per service $0 < c < \infty$. The payoffs are given by (6) below.

In the presence of linear costs, the demand and profit functions may be discontinuous and a Nash equilibrium with pure strategies may not exist in the

[1] The game proposed is a modification of the Economides' (1989) model. This author used a three stage game to compute symmetric equilibrium in a Circular City model, where the first stage is entry, the second location and the third price competition. We simplify the exposition by compacting his first two stages. The use of an additional pool of players does not appear in Economides' model and is an ad hoc addition to ensure the existence of equilibrium. See also Eaton (1976) for a similar model in an unbounded space.

third stage (see D'Asprenmont et al. 1979 and Tirole 1988, chapter 7, for a discussion). One popular solution is to impose a quadratic cost structure. Instead, the linear structure is maintained with the additional assumption that regions can only buy services from their nearest banks. That is, given two banks' locations, all the regions located in between can only buy services at these two places. In equilibrium, it is reasonable to assume that this is true. Moreover, it makes the price discrimination structure consistent.

Sometimes a Salop (1979) Circular City style model is considered more appropriate for modeling strategic interaction. The advantages of this model emerges when considering the Linear City defined on a compact segment [0,1] where locationprice equilibriums cannot be obtained with more than two players (see Gabszewicz and Thisse 1986 and Tirole 1988, chapter 7, for a general discussion). However the interaction properties of my model are similar to those in the Circular City, while an easier treatment of the wealth measure can be done on the real line. The additional pool of banks assumption (or a similar one) would also be required in the Circular City model with an unequal distribution of wealth. The model's goal is to study the distribution of banks, and not the number of entries (which is infinite).

The use of a non-uniform wealth distribution is not new. For instance, Neven (1985) used a two firm Linear City model to study a location-price game. He made use of the same solution concept used here and gave conditions on the distribution density function for obtaining different degrees of differentiation: a higher concentration of wealth produces closer locations. A similar model was developed by Tabuchi and Thisse (1995) for a triangular and symmetric distribution. In this case, no symmetric equilibrium exists. Instead they show the existence of asymmetric equilibriums, characterized by a strong product differentiation. Similar results are obtained by Anderson, Goeree and Ramer (1997). In contrast to those papers, our model allows to handle more than two players.

III. The equilibrium

The equilibrium concept is that of Subgame Perfect Nash Equilibrium (SPNE). For fixed locations (third stage) a Nash equilibrium can be easily found by computing the prices strategies as in a standard Hotelling model. In the second stage, the banks $k=0,1,2,\ldots$ take as given the first-stage locations. No new entrances will occur if the locations in the first stage are sophisticated enough. The location sequence may not be unique, provided that the entire real line is to be filled: Lemma 2 below shows two SPNE location patterns in the uniform case which generate either zero or positive profits (for a discussion about the multiplicity of equilibriums see Eaton 1976). A formal definition of the equilibrium concept is provided:

Definition 1 (Equilibrium): A SPNE of the spatial banking game is given by a sequence $\left\{E_i^*, z_i^*, P_i^{-*}, P_i^{+*}, E_k^*, y_k^*, P_k^{-*}, P_k^{+*}\right\}_{i=0}^{\infty}$ such that for each bank $i = 0,1,2\ldots$ and $k = 0,1,2\ldots$, $\left\{P_i^{-*}, P_i^{+*}\right\}_{i=0}^{\infty}$ and $\left\{P_k^{-*}, P_k^{+*}\right\}_{k=0}^{\infty}$ maximize the first-stage and second-stage bank's profits given in (6) and are non-negative, for fixed locations; $\left\{E_k^*, y_k^*\right\}_{k=0}^{\infty}$ is a Nash equilibrium for the $k = 0,1,2,\ldots$ banks (i.e., each k bank has no incentives to deviate given the strategies of the other $0,1,\ldots k\text{-}1,k+1\ldots$ second-stage banks), where the first stage locations are fixed and prices strategies are optimal in the third stage; and finally $\left\{E_i^*, z_i^*\right\}_{i=0}^{\infty}$ is also a Nash equilibrium of the first stage subgame (i.e., each i bank has no incentives to deviate given the strategies of the other $0,1,\ldots i\text{-}1,i+1\ldots$ first-stage banks), taking into consideration potential entrances in the second-stage and the terminal node price strategies.

A. Third stage: Price competition

Through this subsection we assume that no entries occur in the second-stage, although the description applies to both sets of banks since locations are fixed. Assume a bank in the $z\in\Re$ position of the line-market. It will attract individuals from the left (-) and from the right (+). Let's call these individuals *borrowers*. The scope of each bank is given by those individuals who prefer buying bank services at the z location. Define:

Definition 2: $\left\{P_i^-, P_i^+\right\}$ are the prices that the i^{th} bank, which is in $z_i\in\Re$, charges to its right and left respectively. P_{+i}^- is the (left) price of the nearest bank from the right, which is in $z_{+i}\in\Re$ $(z_{+i} > z_i)$. P_{-i}^+ and z_{-i} $(z_i > z_{-i})$ are the (right) price and location of the nearest bank from the left.

Definition 3: x_i^+ is the distance between bank i (at z_i) and the borrower who is indifferent between buying services in z_i and z_{+i}. x_i^- is the distance between bank i and the individual who is indifferent between buying services in z_i and z_{-i}.

The indifferent borrowers can be found using the following indifference equations:

$$P_i^+ + hx = P_{+i}^- + h\left[(z_{+i} - z_i) - x\right] \tag{1}$$

$$P_i^- + hx = P_{-i}^+ + h\left[(z_i - z_{-i}) - x\right] \tag{2}$$

The bank's scope for both sides can be easily obtained as:

$$x_i^+ = \frac{P_{+i}^- - P_i^+ + h(z_{+i} - z_i)}{2h}, \tag{3}$$

$$x_i^- = \frac{P_{-i}^+ - P_i^- + h(z_i - z_{-i})}{2h} \tag{4}$$

Adding both equations,

$$X_i = x_i^- + x_i^+ = \frac{1}{2h}\left(P_{+i}^- + P_{-i}^+ - P_i^- - P_i^+ + h(z_{+i} - z_{-i})\right) \tag{5}$$

The scope of each bank depends on its price and location, the prices and locations of its rivals and the costs of entry, production and monitoring. Each bank maximizes the following profit function:

$$\Pi_i = (P_i^- - c)\int_{z_i - x_i^-}^{z_i} f(x)dx + (P_i^+ - c)\int_{z_i}^{z_i + x_i^+} f(x)dx - s \equiv (P_i^- - c)Q_i^- + (P_i^+ - c)Q_i^+ - s \tag{6}$$

where $Q_i^- \equiv \int_{z_i - x_i^-}^{z_i} f(x)dx$ and $Q_i^+ \equiv \int_{z_i}^{z_i + x_i^+} f(x)dx$ subject to $P_i^-, P_i^+, x_i^-, x_i^+ \geq 0$. If we assume that the prices and places of all the banks are given, the i^{th} bank will act as a local monopolist.

Assuming an interior solution and using the Leibniz rule, the first order conditions can be expressed as:

$$Q_i^+ - \frac{1}{2h}(P_i^+ - c)f(z_i + x_i^+) = 0, \tag{7}$$

$$Q_i^- - \frac{1}{2h}(P_i^- - c)f(z_i - x_i^-) = 0. \tag{8}$$

The second order condition requires that $-\left(\frac{1}{h}f(z_i + x_i^+) - \frac{P_i^+ - c}{4h^2}f'(z_i + x_i^+)\right) \leq 0$ and $-\left(\frac{1}{h}f(z_i - x_i^-) + \frac{P_i^- - c}{4h^2}f'(z_i - x_i^-)\right) \leq 0$. The bank responds in the usual way to an increment in the distance or price of the rival banks. The following Lemma shows the existence and uniqueness of a Nash equilibrium.

Lemma 1: Consider Assumption 1. For fixed locations, there exists a unique Nash equilibrium in the third stage subgame.

Proof: The existence and uniqueness of a Nash equilibrium has been extensively analyzed for the uniform case (see Gabszewicz and Thisse 1986). The price differentiation setup and the assumption that borrowers can only buy services from their nearest banks allows proving the existence and uniqueness of Nash equilibriums by looking at pair-wise Nash equilibriums of two banks each controlling only one price.

For the non-uniform case, it involves solving a system of two non-linear equations with two unknowns. Without loss of generality assume that the banks i=0 and i=1 are contiguous with locations $z_0 = 0$ and $z_0 = 1$ respectively. As stated above, we assume that v is large enough to generate competition between the two banks (if that is not the case, the banks behave as monopolists, which generates a unique price strategy).

$$g_0(P_0^+, P_1^-) \equiv \int_0^{t(P_0^+, P_1^-)} f(x)dx - \frac{1}{2h}(P_0^+ - c)\, f\left(t(P_0^+, P_1^-)\right) = 0 \tag{9}$$

$$g_1(P_0^+, P_1^-) \equiv \int_{t(P_0^+, P_1^-)}^{1} f(x)dx - \frac{1}{2h}(P_1^- - c)\, f\left(t(P_0^+, P_1^-)\right) = 0 \tag{10}$$

where the two unknowns are $\left(P_0^+, P_1^-\right)$ and $t(P_0^+, P_1^-) = \min\left(\max\left(\frac{-P_0^+ + P_1^- + h}{2h}, 0\right), 1\right)$. Note that both functions are continuous in both arguments. Consider $\tilde{P} = c + 2h\left(\int_0^{1/2} f(x)\right) / \left(f(1/2)\right)$ and let $\overline{P}$ be a large enough price (i.e., v). Define $X_0^+ = \left\{\left(P_0^+, P_1^-\right): P_0^+ \in [c, \tilde{P}), P_1^- \in [\tilde{P}, \overline{P})\right\}$ and $X_0^- = \left\{\left(P_0^+, P_1^-\right): P_0^+ \in [\tilde{P}, \overline{P}), P_1^- \in [c, \tilde{P})\right\}$. Note that for any $\left(P_0^+, P_1^-\right) \in X_0^+$ we have $g_0(.) > 0$ while for $\left(P_0^+, P_1^-\right) \in X_0^-$, $g_0(.) \leq 0$. Also similar arguments can be used to show the existence of the sets X_1^+ and X_1^- which produce positive and non-positive values of $g_1(.)$ respectively. The existence of a solution follows from Miranda's Theorem on the existence of a zero of a nonlinear mapping, which is a generalization of the one-dimensional intermediate value theorem. Uniqueness is a consequence of the monotonicity of $f(.)$.

B. First stage: Location

Define Z, the space of spatial locations, as the set of all possible collection of locations of the form $\{z_j\}_{-\infty}^{\infty}$ with $-\infty < z_{j-1} < z_j < z_{j+1} < \infty\ \forall j$. Let $\{j\}_{-\infty}^{\infty}$ denote a spatial ordering generated by those locations. Since for fixed locations prices are unique, any SPNE can be seen as an element of Z with an induced price sequence.

Define $\Pi\left(z; \underline{z}, \overline{z}\right)$ as the profit obtained, as in (6), by a bank located at $z \in \left(\underline{z}, \overline{z}\right)$ with its closest competitors at $\left(\underline{z}, \overline{z}\right)$. Also define $\Delta(\underline{z}, z) = \arg\sup_z \Pi\left(z; \underline{z}, \overline{z}\right)$.

Consider the following definitions:

Definition 4: $\{z_j\}_{-\infty}^{\infty}$ is a *profitable entry-deterrence* sequence if it is a spatial location such that $\Pi(z_j; z_{j-1}, z_{j+1}) \geq 0 \quad \forall j$ and $\Pi(\Delta(z_j, z_{j+1}); z_j, z_{j+1}) \leq 0 \quad \forall j$.
$\{z_j\}_{-\infty}^{\infty}$ is a *maximum differentiation* sequence if it is a spatial location such that $\Pi(z_j; z_{j-1}, z_{j+1}) \geq 0 \quad \forall j$ and $\Pi(\Delta(z_j, z_{j+1}); z_j, z_{j+1}) = 0 \quad \forall j$.
$\{z_j\}_{-\infty}^{\infty}$ is a *zero-profit* sequence if it is a spatial location such that $\Pi(z_j; z_{j-1}, z_{j+1}) = 0 \quad \forall j$ and $\Pi(\Delta(z_j, z_{j+1}); z_j, z_{j+1}) \leq 0 \quad \forall j$.

A profitable entry-deterrence sequence does not have any entry in the secondstage. Also note that maximum differentiation and zero-profit sequences are special cases of the former.

Uniform distribution of wealth

Lemma 2: Consider a uniform distribution of wealth (i.e., $f(x) = 1$ for all x). Assume that $v \geq \sqrt{\frac{s}{2h}}$. Let $\{j\}_{-\infty}^{\infty}$ denote a spatial ordering. Then any symmetric (i.e., $z_j^* - z_{j-1}^* = z_{j+1}^* - z_j^*$) profitable entry-deterrence sequence is a SPNE. Consequently a zero-profit SPNE exists and has a location sequence satisfying $z_{j+1}^* - z_j^* = \sqrt{\frac{s}{h}} \quad \forall j$ and a maximum differentiation SPNE exists and has a location sequence $z_{j+1}^* - z_j^* = \sqrt{\frac{2s}{h}} \quad \forall j$.

Proof: In a uniform distribution, any new entrant (k) will locate at $y_k = \frac{z_{j+1}^* + z_j^*}{2}$ Moreover, prices will be symmetric (i.e., $P = P_k^+ = P_k^- = P_j^{+*} = P_{j+1}^{-*}$) and profits for bank k are a non-decreasing function of the distance $d = z_{j+1}^* - z_j^*$. Moreover by symmetry of the uniform case the entrant bank will satisfy $d/2$. Entry deterrence is obtained by finding d which satisfies

$$\frac{d}{2} + \left(-\frac{1}{2h}\right)(P - c) = 0, \tag{11}$$

$$\frac{d}{2}(P - c) = s, \tag{12}$$

where the first equation corresponds to the first order condition, taking into consideration the potential (symmetric) effect on their competitors and the second equation is the zero-profit condition. Solving for d, we get 2s $d = \sqrt{\frac{2s}{h}}$. Therefore no additional entrances would occur in the second stage for a first-stage bank location sequence such that $\max\{z_j^* - z_{j-1}^*, z_{j+1}^* - z_j^*\} \leq \sqrt{\frac{2s}{h}} \quad \forall j$.

In a maximum differentiation SPNE we have $z_{j+1}^* - z_j^* = \sqrt{\frac{2s}{h}} \quad \forall j$, which generates a profit sequence $\{\Pi_j^* = s\}_{-\infty}^{\infty}$. A zero-profit SPNE is obtained by setting

$z^*_{j+1} - z^*_j = \sqrt{\frac{s}{h}} \quad \forall j$. Note that banks have no incentives to change location, provided that the remaining banks are immobile, and symmetric differentiation $z^*_j - z^*_{j-1} = z^*_{j+1} - z^*_j$ is optimal in the uniform case.

Non-uniform distribution of wealth

Unlike the uniform case, in a non-uniform wealth distribution banks may prefer to move towards higher density regions, for fixed locations of their competitors. In turn this leapfrogs those banks that may initially choose to settle in a high density zone, destroying the equilibrium. This may be avoided by the proposed sequential location structure, where additional banks enter the market after the first round of locations is done. If, for fixed competitor's locations, a bank decides to change its place, other banks would enter the market, significantly reducing the benefits of moving.

Definition 5: $\{z^*_j\}_{-\infty}^{\infty}$ is a *location compatible* sequence if it is a spatial location such that $\Pi^*_j \equiv \Pi(z^*_j; z^*_{j-1}, z^*_{j+1}) \geq 0 \quad \forall j$; $\Pi(\Delta(z^*_j, z^*_{j+1}); z^*_j, z^*_{j+1}) = 0 \quad \forall j$ and

$$\max\left\{\sup_{z_j} \Pi(z_j; z^*_{j-1}, \Delta(z_j, z^*_{j+1})) \sup_{z_j} \Pi(z_j; \Delta(z^*_{j-1}, z_j), z^*_{j+1},)\right\} \leq \Pi^*_j \quad \forall j. \tag{13}$$

Condition (13) implies that when considering the remaining (first-stage) banks' location as fixed, no deviation, which attracts a new entrance, would produce a profit higher than the one obtained at equilibrium. This condition is satisfied in the uniform distribution equilibriums described in Lemma 2.

Lemma 3: Consider a non-uniform distribution (as in Assumption 1). Assume that $v \geq \sqrt{\frac{s}{2h}}$ Then, any location compatible sequence is SPNE.

Proof: It is only needed to show that in the first stage, for fixed locations of the other banks, no bank has incentives to move. Consider bank *j*, satisfying $z^*_{j-1} < z^*_j < z^*_{j+1}$, with profit level Π^*_j (defined in Definition 5), and located in the nonuniform density region. Suppose that a new location is preferable (say $0 < z^*_{j-1} < z_j < z^*_j < z^*_{j+1}$, where the bank *j* prefers to move to a higher density zone). This motivates the entrance of at least one more bank in the second stage (for more than one bank a similar argument applies). If only one bank enters in the second stage (say *k*), it will be located to the right of the *j*-bank: $z^*_{j-1} < z_j < y_k < z^*_{j+1}$. If $y_k \leq z^*_j$, clearly the new location of bank *j* would generate negative profits (by the entry-deterrence condition), then $y_k > z^*_j$. Moreover by assumption, $\sup_y \Pi(y; z_j, y_k) \leq 0$ (only one additional bank). Then, if z_j becomes closer to z^*_{j-1}.

profits became non-positive (since y_k goes to z_j^*, which implies that profits increase in the opposite direction. As z_j approaches z_j^*, profits are below the original level (since there is an additional bank k). The location compatible property (13) implies that $\sup_{z_j} \Pi\left(z_j; z_{j-1}^*, \Delta(z_j, z_{j+1}^*)\right) \leq \Pi_j^*$, therefore the profit level cannot be above the original one, which contradicts that the new location was better.

Note that a zero-profit equilibrium cannot be achieved in this context. If a bank deviates from its original location (i.e. moving towards the high density regions), it can do so without attracting new entrances. Moreover maximum differentiation is not enough to ensure the existence of SPNE.

Also note that as $|x| \to \infty$ and $f(x) \to 1$ the location sequence becomes that described in Lemma 2. We will use the term "convergence" to denote the fact that a location sequence becomes that of the uniform case as $|x| \to \infty$. Note that if the location sequence converges, prices and profits will also converge to the uniform case. We will say that a sequence converges faster than other sequence, if smaller value of |x| is required to observe convergence to the uniform case.

C. Simulation and empirical implications

Unfortunately, in general the model does not provide explicit analytical solutions for a non-uniform distribution of wealth. The reason is that solving the first order conditions requires solving a system of two non-linear equations with two unknowns, since this involves using both the density and the distribution functions (see Lemma 1 for a discussion and existence of solution).

To study the distribution pattern of banks across regions we use the following non-uniform distribution of wealth:

$$f(x) = \phi(x;\sigma) + 1, \tag{14}$$

where $\phi(x;\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$, that is a normal density function with mean zero and standard deviation σ.[2] It can be easily checked that $f(x)$ satisfies Assumption 1, that is, it is symmetric about zero and converges to 1 as x goes to infinity.

In this subsection it is showed that in equilibrium banks are more concentrated in richer regions and more disperse in poorer ones. Therefore, richer regions will face lower prices. Moreover, the average monitoring cost in a poor region will be higher than the one in a rich region, provided that for the same interval length,

[2] Similar results were obtained for other unimodal distributions.

more banks will be located in the latter than in the former. Comparative statics results where we consider changes in the monitoring costs (h) and the concentration of wealth (σ) are provided in the following subsections.

To show how the model works, consider a simulation with parameters $c = 1$, $s = 1.25$, $h = 10$, $\sigma = 1$.[3] In equilibrium, as x goes to infinity and the wealth distribution becomes uniform we will obtain equal spacing of banks, with $d_U = \sqrt{\frac{2s}{h}} = 0.5$; profits will be $\Pi_U = s = 1.25$ and prices $P_U = 2s/d + c = 6$ (see Lemma 2). In other words, the distance between contiguous banks, prices and profit sequences will converge to the uniform case. The following paragraphs describe the simulation exercise.

Since the distribution of wealth is unimodal, assume that a bank is located at the mode. This requirement is not necessary (the equilibrium is not unique) but it simplifies the computation. Let this bank be $i = 0$ (then $z_0 = 0$). Moreover, let the banks located in the positive locations be odd, and in the negative be even.
Therefore $z_1 \in (0,\infty)$, $z_2 \in (-\infty,0)$, $z_i > z_1$ for i odd and $z_2 > z_i$ for i even. Consider first the location of the $i = 1$ bank. Entry deterrence is achieved by any location $0 < z_1 \le z_1^*$, where $z_1^* = \sup_z \left\{ z > 0 : \sup_x \Pi(x;0,z) \le 0 \right\}$. To achieve a *location compatible sequence* we set $z_1 = z_1^*$. We use a simulation exercise to compute z_1^* based on the following algorithm:

- Construct a grid spacing of 0.005, i.e. $\Omega_1 = \{0,\ 0.005,\ 0.01,\ 0.015, \dots\}$
- For each $z \in \Omega_1$, construct $x(z) = \left\{ x \in \Omega_1 : \max_x \Pi(x;0,z) \right\}$
- Define $z_1^* = \max_z \left\{ z \in \Omega_1 : \Pi(x(z);0,z) \le 0 \right\}$

The same process can be applied to $i = 3$, by choosing $z_3^* = \sup_z \left\{ z > z_1^* : \sup_x \Pi(x; z_1^*, z) \le 0 \right\}$ and to higher odd-numbered banks. In that case we use the following algorithm, for $i = 3,5,7\dots$:

- Construct a grid spacing of 0.005, i.e. $\Omega_1 = \{z_{i-2}^*, z_{i-2}^* + 0.005, z_{i-2}^* + 2\times 0.005, z_{i-2}^* + 3\times 0.005, \dots\}$
- For each $z \in \Omega_i$, construct $x(z) = \left\{ x \in \Omega_i : \max_x \Pi(x; z_{i-2}^*, z) \right\}$
- Define $z_i^* = \max_z \left\{ z \in \Omega_i : \Pi(x(z); z_{i-2}^*, z) \le 0 \right\}$

By symmetry, set $z_i^* = -z_{i+1}^*$ for $i = 1,3,5\dots$ Condition (13) and the second order conditions of the third stage (see footnote 2) need to be checked on a case by case basis.

[3] A high value of h is used to have a significant location sequence in the non-uniform part of the wealth measure. Similar results may be achieved by expanding the density mass (i.e. multiplying by a constant).

Figure 1A shows the computed location and distance between contiguous banks, that is, the graph $\left\{\left(z_i^*, z_{i+2}^* - z_i^*\right) : i = 0,1,3,5....\right\}$. It can be noted that banks become more disperse as the distribution of wealth becomes more uniform. Therefore, high wealth density regions (e.g., an interval around $x = 0$) will face lower average monitoring costs as more banks are located there than in any other interval of the same length in a uniformly distributed region. Figure 1B plots the price sequences $\left\{\left(z_i^*, P_i^{-*}\right), \left(z_i^*, P_i^{+*}\right) : \text{i=0,1,3,5...}\right\}$. Prices increase monotonically as x increases. Hence, high wealth-density regions will also face lower prices. The intuition behind the model can be summarized as follows: in regions with high wealth, (first stage) banks are compelled to come closer together to avoid future entrances (in the second stage), but this results in more competition, and consequently lower prices. Also, note that there exist small differences between P^{-*} and P^{+*}: for $i = 0$ both prices are equal, and we have that $P_i^{-*} \leq P_i^{+*}$, i=0,1,3,5... . The reason is that for $x > 0$ (i.e., i odd), P^- represents the price charged towards the higher density region, and by lowering the price, the bank will be able to attract more wealthy borrowers. At the same time, each bank faces more competition towards the higher density direction. Finally Figure 1C reports the profit sequence $\left\{\left(z_i^*, \Pi_i^*\right) : i = 0,1,3,5....\right\}$. In this case, no clear pattern emerges: if on the one hand, more competition translates into lower prices (and lower profits), on the other hand more density of wealth produces larger profits.

Figure 1A. Location and distance between contiguous banks

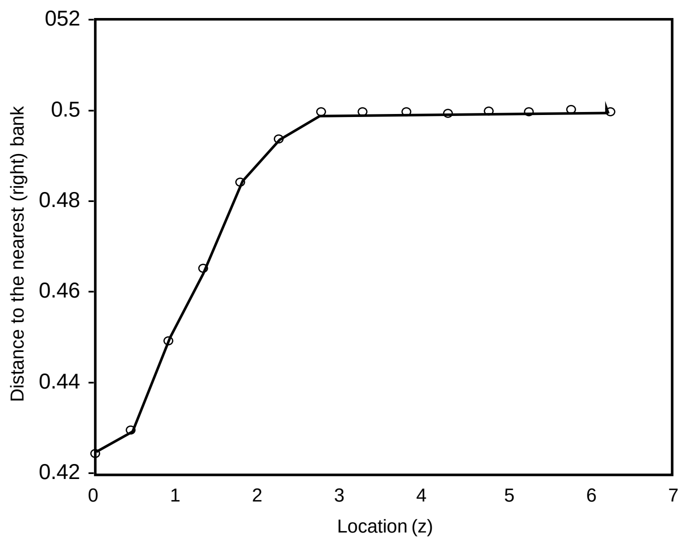


Figure 1B. Location and prices

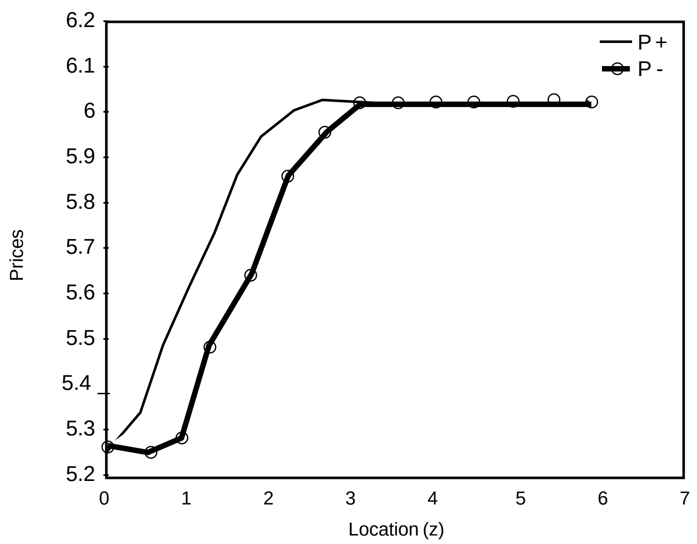
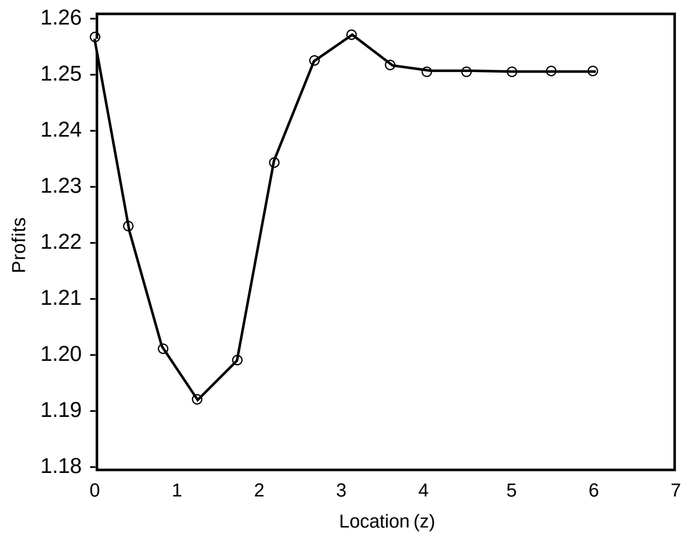


Figure 1C. Location and profits



This model is in line with the Berger and Hannan (1989) price-concentration relationship. Those authors found that prices are less favorable to consumers in more concentrated markets (regions with more dispersed banks in our case) because of the non-competitive behavior. Moreover, the fact that regions with lower average monitoring costs (i.e., a measure of the average distance to the nearest bank) face lower prices corresponds to the Petersen and Rajan (2002) findings. Furthermore, except for the bank located at $x = 0$, the average profits are lower in regions with high wealth density (i.e. $\Pi < 1.25$) than in regions with low wealth density ($\Pi_U = 1.25$). This is consistent with Pilloff and Rhoades's (2002) profitconcentration relation and with Burdisso and D'Amato's (1999) results for Argentina.

The results of the model contribute to the understanding of regional disparities in the banking industry. In this case, a rich region should be associated with high values of *f*(.), while a poor region with low values of *f*(.). In the former, the concentration of wealth (i.e. more wealthy clients or more clients) attracts more banks, which produces lower average monitoring costs and lower prices than in the latter.

D. Information technologies

The model provides a simple framework to study the effect of the introduction of a new technology (a reduction in *h*, the monitoring cost). As Lemma 2 shows, in equilibrium with uniform wealth, the distance between contiguous banks is proportional to $\sqrt{\frac{s}{h}}$. Moreover prices would be proportional to *h*, that is $P(h) = c + hd(h) = c + A\sqrt{sh}$,where *A* is a constant which depends on the SPNE used but not on *h*. Consequently, a reduction in *h* would produce higher dispersion among banks but lower prices. A priori, we expect a similar effect to appear in the non-uniform case. For the non-uniform case, we provide simulations to study the effect of a change in *h*. We construct simulations as in Section III.C where *h* takes values in {6,8,10}. Figures 2A through 2C report the graphs

$$\left\{\left(z_i^*(h), z_{i+2}^*(h) - z_i^*(h)\right) : i = 0,1,3,5....,h = 6,8,10\right\} \tag{15}$$

$$\left\{\left(z_i^*(h), P_i^{-*}(h)\right), \left(z_i^*(h), P_i^{+*}(h)\right) : \text{i=0,1,3,5...,h=6,8,10}\right\} \tag{16}$$

$$\left\{\left(z_i^*(h), \Pi_i^*(h)\right) : i = 0,1,3,5....,h = 6,8,10\right\} \tag{17}$$

The simulations show similar results to those obtained in the uniform case, that is, as *h* decreases (i.e. better monitoring technology), the distance between banks increases, prices decrease and there is no clear effect on profits.

Figure 2A. Location and distance: Different information technologies

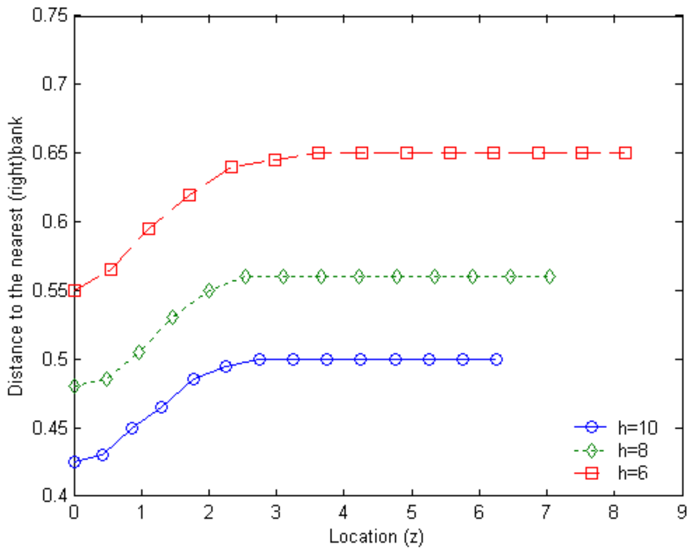


Figure 2B. Location and prices: Different information technologies

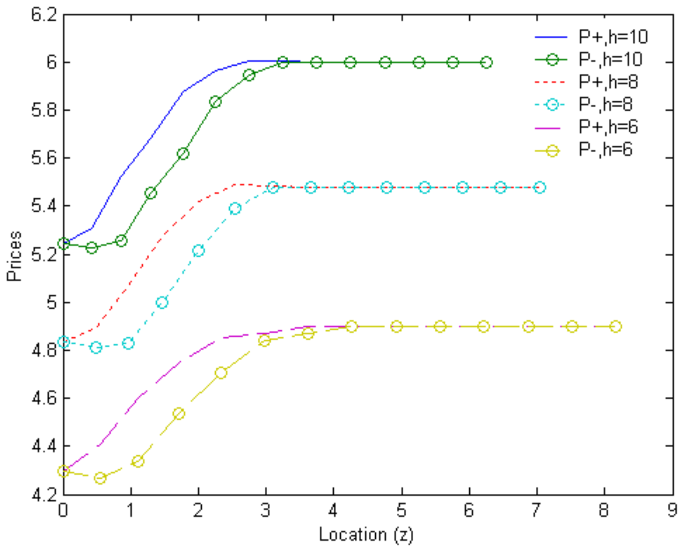
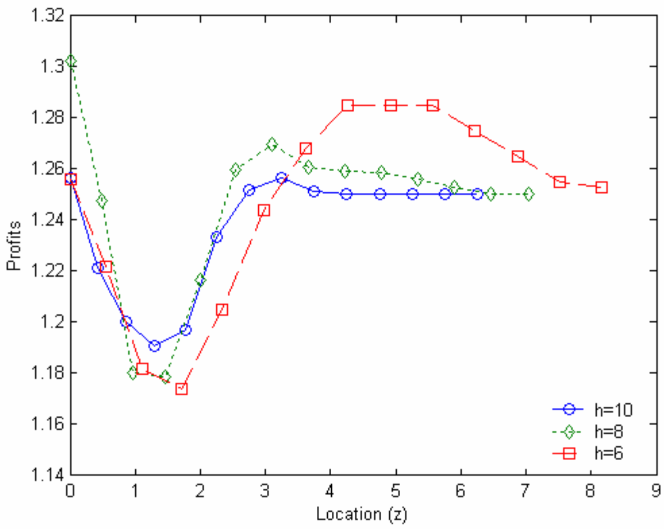


Figure 2C. Location and profits: Different information technologies



E. Concentration of wealth

As a final exercise we study the effect of different concentration of wealth, as measured by different values of the dispersion parameter. Again, we construct simulations as in Section III.C where σ takes values in {1,2,3}. Figures 3A through 3C report the graphs

$$\left\{\left(z_i^*(\sigma), z_{i+2}^*(\sigma)-z_i^*(\sigma)\right): i=0,1,3,5....,\sigma=1,2,3\right\}, \tag{18}$$

$$\left\{\left(z_i^*(\sigma), P_i^{-*}(\sigma)\right),\left(z_i^*(\sigma), P_i^{+*}(\sigma)\right): \text{i=0,1,3,5...,}\sigma\text{=1,2,3}\right\}, \tag{18}$$

$$\left\{\left(z_i^*(\sigma), \Pi_i^*(\sigma)\right): i=0,1,3,5....,\sigma=1,2,3\right\}. \tag{20}$$

We observe that the smaller the value of dispersion, the faster the convergence to the uniform case distance among contiguous banks (= 0.5) and prices (= 6). Regions near x = 0 would face lower average distance and prices for a smaller value of σ. However borrowers located before the convergence to the uniform case but not close enough to x = 0, will prefer higher values of σ. The same pattern is observed for profits, that is, faster convergence for smaller values of σ. In this case, for σ = 3, the range considered is not enough to observe convergence.

Figure 3A. Location and distance: Different concentration of wealth

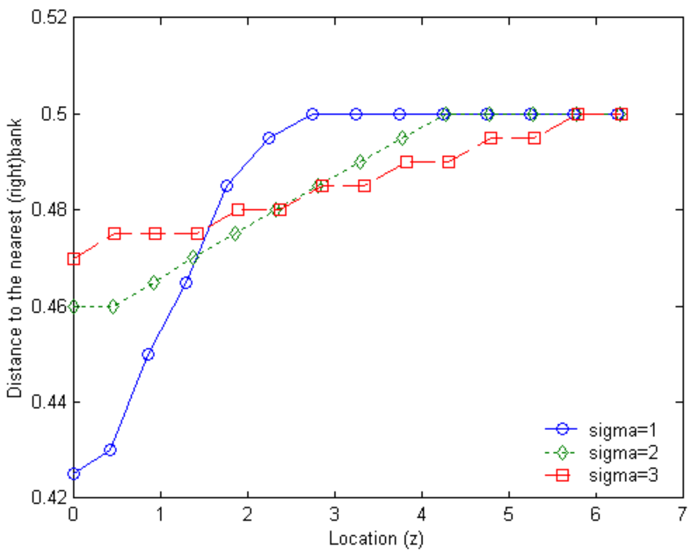


Figure 3B. Location and prices: Different concentration of wealth

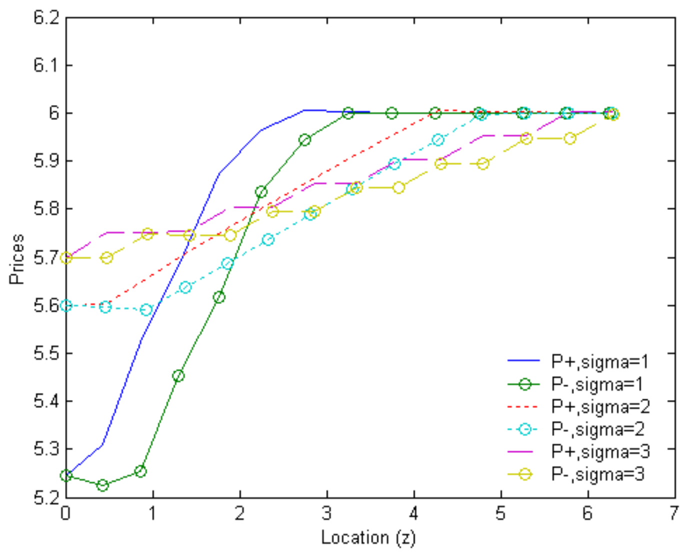
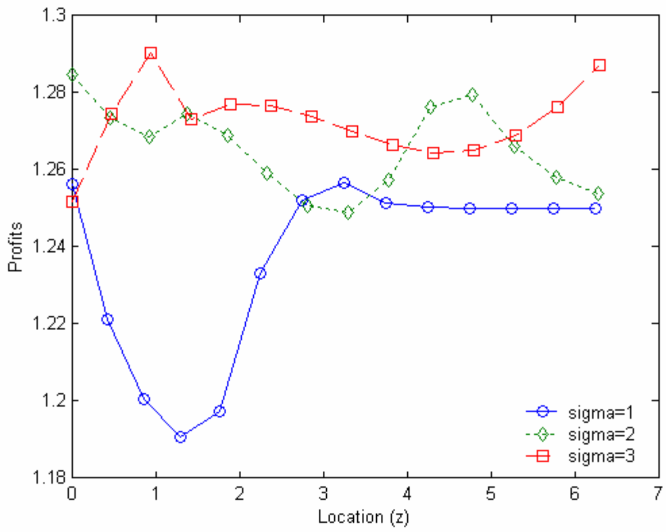


Figure 3C. Location and profits: Different concentration of wealth



IV. Conclusions

The paper explored the importance of a non-uniform wealth distribution in a spatial banking model. Without loss of generality we consider a unimodal distribution of wealth. In a free entry model, the banks will be unequally distributed across regions if wealth also is. Richer regions will have more banks, lower prices and a lower mean monitoring cost than poorer regions. In turn, this exacerbates regional disparities, provided that access to financial services contributes to the regional growth potential.

Additionally it studies the effect of the introduction of better technologies in the spatial location of banks. As in the uniform case, lower monitoring costs produces more differentiation, but lower prices. We also study changes in the concentration of wealth across regions. In this case, regions close enough to the mode of the wealth distribution would face the lowest average distance and prices for small values of the dispersion parameter. However, this does not apply if we consider regions distant from the mode.

The model can also be applied to other industries that share the same logic, where the customers are responsible for paying a cost proportional to the "distance" to the firm and the firm has some local monopoly power over its closest clients. The model predicts that the distribution of wealth influences the location pattern of firms, their prices and profits, and the average monitoring/transportation cost customers in different regions may face.

References

Anderson, Simon P., Jacob K. Goeree and Roald Ramer (1997) "Location, location, location," *Journal of Economic Theory* **77**: 102-27.

Berger, Allan N., and Timothy H. Hannan (1989), "The price-concentration relationship in banking", *Review of Economics and Statistics* **71**: 291-99.

Boot, Arnoud W.A., and Anjan V. Thakor (1997), "Banking scope and financial innovation", *Review of Financial Studies* **10**: 1099-131.

Burdisso, Tamara, and Laura D'Amato (1999), "Prudential regulation, restructuring and competition: The case of the Argentine banking industry", Working Paper 10, Buenos

Aires, Banco Central de la República Argentina. Chiappori, Pierre-Andre, David Perez-Castillo, and Thierry Verdier (1995), "Spatial competition

in the banking system: Localization, cross subsidies and the regulation of deposit rates", *European Economic Review* **39**, 889-918.

D'Asprenmont, C., Jean J. Gabszewicz, and Jacques-François Thisse (1979), "On Hotelling's 'stability in competition'", *Econometrica* **47**: 1145-50.

Dell'Ariccia, Giovanni (2001), "Asymmetric information and the structure of the banking industry", *European Economic Review* **45**: 1957-80.

Eaton, B. Curtis (1976), "Free entry in one-dimensional models: Pure profits and multiple equilibria", *Journal of Regional Science* **16**: 21-33.

Economides, Nicholas S. (1989), "Symmetric equilibrium existence and optimality in differentiated product markets", *Journal of Economic Theory* **47**: 178-94.

Gabszewicz, Jean J., and Jacques-François Thisse (1986), "Spatial competition and the location of firms", in J.J. Gabszewicz, J.-F. Thisse, M. Fushita and U. Schweizer, eds., *Location theory*, Chur, Switzerland, Harwood Academic Publishers GmbH.

Hannan, Timothy H. (1991), "Bank commercial loan markets and the role of the market structure: Evidence from surveys of commercial lending", *Journal of Banking and Finance* **15**: 133-49.

Hauswald, Robert, and Robert Marquez (2004) "Information technology and financial service competition", *Review of Financial Studies* **16**: 921-48.

Hotelling, Harold (1929), "The stability of competition", *Economic Journal* **39**: 41-57 Kashyap, Anil K., Raghuran G. Rajan and Jeremy C. Stein (2002) 'Banks as liquidity providers: an explanation for the coexistence of lending and deposit taking", *Journal of Finance* **57**: 33-73.

Keeton, William R. (1998), "Are rural banks facing increased funding pressures? Evidence from Tenth District states", *Federal Reserve Bank of Kansas City Economic Review* **83**: 43-67.

Marquez, Robert (2002), "Competition, adverse selection, and information dispersion in the banking industry", *Review of Financial Studies* **15**: 901-26.

Neven, Damien J. (1986), "On Hotelling's competition with non-uniform customer distribution", *Economics Letters* **21**: 121-26.

Petersen, Mitchell A., and Raghuran G. Rajan (2002) "Does distance still matter? The information revolution in small business lending", *Journal of Finance* **57**: 2533-70.

Pilloff, Steven J., and Stephen A. Rhoades (2002), "Structure and profitability in banking markets", *Review of Industrial Organization* **20**: 81-98.

Rajan, Raghuran G. (1992), "Insiders and outsiders: The choice between informed and arm'slength debt", *Journal of Finance* **47**: 1367-400.

Rajan, Raghuran G. (1998) "The past and future of commercial banking viewed through an incomplete contract lens", *Journal of Money, Credit and Banking* **30**: 524-50.

Salop , Steven C. (1979), "Monopolistic competition with outside goods", *Bell Journal of Economics* **10**: 141-56.

Tabuchi, Takatoshi, and Jacques-François Thisse (1995), "Asymmetric equilibria in spatial competition", *International Journal of Industrial Organization* **13**: 213-27.

Tirole, Jean (1988), *The Theory of Industrial Organization*, Cambridge, MA, MIT Press.

Wilhelm, William J. (1999), "Internet investment banking: The impact of information technology on relationship banking", *Journal of Applied Corporate Finance* **12**: 21-27.

Wong, Kit Pong, and Derek K. Chan (1993), "A simple model of spatial banking competition", *Economics Letters* **42**: 391-97.