
ELSEVIER
Volume 140
Issue 4
April 2010
ISSN 0378-3758
journal of statistical planning and inference
For Contents see page iii
Available online at www.sciencedirect.com
ScienceDirect

Journal of Statistical Planning and Inference 140 (2010) 1013–1020

Contents lists available at ScienceDirect

Journal of Statistical Planning and Inference

journal homepage: www.elsevier.com/locate/jspi

# Testing for random effects and serial correlation in spatial autoregressive models

Gabriel V. Montes-Rojas *

*Department of Economics, City University London, Northampton Square, London EC1V 0HB, UK*

A R T I C L E   I N F O

*Article history:*
Received 14 August 2008
Received in revised form
5 October 2009
Accepted 7 October 2009
Available online 14 October 2009

*JEL classification:*
C12
C23

*Keywords:*
Spatial autoregressive
Random effects
Serial correlation
LM tests

A B S T R A C T

This paper constructs and evaluates tests for random effects and serial correlation in spatial autoregressive panel data models. In these models, ignoring the presence of random effects not only produces misleading inference but inconsistent estimation of the regression coefficients. Two different estimation methods are considered: maximum likelihood and instrumental variables. For each estimator, optimal tests are constructed: Lagrange multiplier in the first case; Neyman's $C(\alpha)$ in the second. In addition, locally size-robust tests, for individual hypotheses under local misspecification of the unconsidered parameter, are constructed. Extensive Monte Carlo evidence is presented.



## 1. Introduction

This paper considers an extension of the Bera et al. (2001) specification tests for random effects and serial correlation in panel data models to spatial autoregressive (SAR) models. SAR panel data models have a distinctive feature that enhances the importance of these tests: they do not have block diagonality in the information matrix. In consequence, misspecification in one component may result in inconsistent estimates of other parameters. In particular, the presence of random effects has a first order effect on the estimation of the regression coefficients, and therefore, testing for it is an important issue. We consider Lagrange multiplier (LM) tests, which have the advantage that the parameters in the null hypothesis need not be estimated. These tests are complementary to the work of Baltagi et al. (2003, 2007) that constructed similar tests for a panel data model with spatial dependence in the error component.

Bera et al. (2001) show that marginal tests for either random effects or serial correlation have incorrect asymptotic size under local misspecification of the unconsidered parameter. In particular, they find that the presence of first order serial correlation makes the standard Breusch and Pagan (1980) test for random effects reject the null hypothesis too often, thus implying that rejections of the null may be due to the presence of random effects but also due to the presence of first order serial correlation. Moreover, similar over-rejections occur for the test for first order serial correlation when random effects are ignored. They develop a procedure for constructing Bera and Yoon (1993) locally size-robust tests, which allows distinguishing the source(s) of misspecification in the null model. We also construct these locally size-robust tests in the SAR framework and we show that they have similar power and better size than standard LM tests.

* Tel.: +44 20 7040 8919.
*E-mail address:* Gabriel.Montes-Rojas.1@city.ac.uk

0378-3758/$ - see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.jspi.2009.10.001

In addition, following the work of Kelejian and Prucha (1997), instrumental variables (IV) estimators are extensively used to estimate SAR models as an alternative to the computationally demanding maximum likelihood (ML) estimator. In general, specification testing procedures developed for panel data and spatial models are based on ML, where LM tests are optimal. This paper applies optimal testing procedures based on Neyman's $C(\alpha)$ tests, which are optimal for any $\sqrt{n}$-consistent estimator (e.g. IV), and compare them to the LM procedures.

The paper is organized as follows. Section 2 presents the panel data SAR model with random effects and first order serial correlation. Section 3 derives LM and $C(\alpha)$ tests. Appendices A and B contain the technical details. Section 4 presents Monte Carlo evidence on the small sample performance of these tests. Conclusions and suggestions for future research appear in Section 5.

## 2. SAR models with random effects and serial correlation

Consider a spatial structure with $N$ regions (denoted by the index $i = 1, 2, \ldots, N$). Each region contains $T$ ($t = 1, 2, \ldots, T$ indexes time) realizations of some random variables. Let $W = \{w_{ij}\}, w_{ij} \geq 0, w_{ii} = 0, \sum_{i \neq j} w_{ij} = 1$ denote a matrix of spatial weights, which summarizes the effect of neighboring regions.[1]

The following regression model adds spatial dependence in the form of a spatial lag to a panel data structure with random effects and first order serial correlation (for SAR models with panel data see Anselin and Bera, 1998; Elhorst, 2003):

$$y_{it} = \tau \sum_{i \neq j}^{N} w_{ij} y_{jt} + x_{it}' \beta + u_{it}, \tag{1}$$

where the disturbances structure is $u_{it} = \mu_i + v_{it}$ and $v_{it} = \rho v_{i,t-1} + e_{it}$. $y_{it}$ is the observation of the dependent variable on the $i$ th region for the $t$ th time period, $x_{it}$ denotes the $k \times 1$ vector of observations on the non-stochastic regressors and $u_{it}$ is the corresponding unobserved error term. Spatial dependence is given by the lag dependence factor $\tau$, $|\tau| < 1$, which provides a measure of the effect of the neighboring regions on the $i$ th region. Additionally $\mu_i \sim i.i.d.\mathrm{N}(0, \sigma_\mu^2)$ is the random effect component of the $i$ th region, $v_{it}$ is the serially correlated component in the $i$ region, and $e_{it} \sim i.i.d.\mathrm{N}(0, \sigma_e^2)$ is the i.i.d. random component. The parameter $\rho$, $|\rho| < 1$, measures the serial autocorrelation for each region.[2]

Model (1) can be rewritten in matrix notation as

$$Y = \tau(I_T \otimes W)Y + X\beta + u \equiv \tau W_\otimes Y + \mathbf{X}\beta + u, \tag{2}$$

where $u = (\iota_T \iota_T' \otimes diag(\mu_1, \mu_2, \ldots, \mu_N))\iota_{NT} + (I_T \otimes I_N)v$. Here $Y$ is of dimension $NT \times 1$, $X$ is $NT \times k$, $\beta$ is $k \times 1$, $u$ is $NT \times 1$, $v' = (v_1', v_2', \ldots, v_T')$ ($v_t$ being a $N \times 1$ vector), $\iota_T$ is a T-dimensional vector of ones, $I$ is an identity matrix, $W$ is the spatial matrix defined above, $\otimes$ denotes the Kronecker matrix product and $W_\otimes = I_T \otimes W$.

Let $B(\tau) = I_N - \tau W$ and $B_\otimes(\tau) = I_T \otimes B(\tau)$. As shown in Anselin and Bera (1998), the covariance of $u$ is $B_\otimes(\tau)^{-1} \Omega(\sigma_e^2, \sigma_\mu^2, \rho) B_\otimes(\tau)^{-1'}$, where $\Omega(\sigma_e^2, \sigma_\mu^2, \rho)$ is the standard variance–covariance matrix of a panel data model with random effects and serial correlation (see for instance Bera et al., 2001). Note that this covariance structure presumes the absence of spatial correlation across time, i.e. the future effect of a given region on its neighbors.

The log likelihood function is

$$\ell(\beta, \tau, \sigma_e^2, \sigma_\mu^2, \rho) = \ln|B_\otimes(\tau) \cdot \Omega(\sigma_e^2, \sigma_\mu^2, \rho)^{1/2}| - \frac{1}{2} u(\beta, \tau)' \Omega(\sigma_e^2, \sigma_\mu^2, \rho)^{-1} u(\beta, \tau), \tag{3}$$

where $u(\beta, \tau) = B_\otimes(\tau)y - \mathbf{X}\beta$. The restricted ML estimator under $H_0^{\sigma_\mu^2 \rho}$ is obtained by maximizing this function with respect to $(\beta, \tau, \sigma_e^2)$, assuming $\sigma_\mu^2 = \rho = 0$.

An alternative to the ML estimator is given by Kelejian and Prucha (1997) estimator. The fact that $Wy$ is correlated with the error term $u$ suggests that *instrumental variables* (IV) can be used to solve for this endogeneity issue. In particular, spatially lag explanatory variables $(W\mathbf{X}, W^2\mathbf{X}, \ldots)$ can be used as IV. We use $W\mathbf{X}$ in our Monte Carlo simulations below.

## 3. LM and C($\alpha$) tests

Our objective is to develop a joint test $H_0^{\sigma_\mu^2 \rho} : \sigma_\mu^2 = \rho = 0$, and marginal tests $H_0^{\sigma_\mu^2} : \sigma_\mu^2 = 0$ and $H_0^{\rho} : \rho = 0$. Under $H_0^{\sigma_\mu^2 \rho}$ the model is the standard SAR. All the tests constructed in this paper are based on the estimation of $(\beta, \tau, \sigma_e^2)$ under the joint null $H_0^{\sigma_\mu^2 \rho}$, i.e. without estimating $\sigma_\mu^2$ or $\rho$.[3]

[1] Note that this matrix need not be symmetric, that is, the effect of a given region on its neighbor may not be the equal to the reciprocal effect.

[2] It is also assumed that $v_0 \sim i.i.d.\mathrm{N}(0, \sigma_e^2/(1-\rho^2))$.

[3] A common practice when there are strong preferences for a restricted model is to start with a small model and then check whether particular departures from this null model are supported or rejected by the data. The main advantage of LM-type statistics is that they are based on the estimation of a full null model, and therefore, tests for individual components do not require estimation of the nuisance parameters. Each marginal LM test is therefore intended to detect departures from a joint null model.

Appendix A derives the score functions and the Jacobian of model (3). A distinctive feature of this model is that the Jacobian is not block diagonal as in Baltagi et al. (2007) where the validity of $H_0^{\sigma_\mu^2\rho}$ does not affect the consistency of the $\beta$ estimator. Here misspecification in $\sigma_\mu^2$ has a potential effect on the estimates of $\beta$. Using the score functions from Appendix A, Appendix B derives the LM tests for the hypotheses of interest. The test for $H_0^{\sigma_\mu^2\rho}$ is defined as $LM_{\sigma_\mu^2\rho}$, a marginal test for $H_0^{\sigma_\mu^2}$ is $LM_{\sigma_\mu^2}$ and for $H_0^\rho$ is $LM_\rho$. Under the joint null hypothesis, $LM_{\sigma_\mu^2\rho}$ has an asymptotic central $\chi_2^2$ distribution; and both $LM_{\sigma_\mu^2}$ and $LM_\rho$ follow a central $\chi_1^2$ distribution.

LM tests are optimal[4] for ML estimators but not necessarily for other consistent estimators (for our purposes, Kelejian and Prucha, 1997, IV estimator). Neyman $C(\alpha)$ tests recover optimality with a simple transformation of the score functions (see Bera and Bilias, 2001). These are defined as $C_{\sigma_\mu^2\rho}$, $C_{\sigma_\mu^2}$ and $C_\rho$ for the joint and each marginal, respectively (see Appendix B). These are expected to have greater power than the LM statistics when the IV estimator is used.

Bera et al. (2001) show that the marginal tests lose the centrality of the $\chi_1^2$ distribution because of model misspecification in the parameters not being tested (e.g. testing for random effects assuming no serial correlation where, in fact, the error term is serially correlated, and vice versa), and this produces spurious rejections. Those authors follow Bera and Yoon (1993) procedure and construct adjusted $LM$ tests to recover the centrality property under local misspecification (of order $O(1/\sqrt{NT})$). These adjusted tests have the same asymptotic distribution (i.e. central $\chi_1^2$) that the corresponding marginal tests under the joint null hypothesis and in the presence of local misspecification in the parameters not being tested. Following Bera et al. (2001), first, we construct tests for $H_0^{\sigma_\mu^2}$ that are size-robust to local misspecification in $H_0^\rho$, $LM_{\sigma_\mu^2(\rho)}$ and $C_{\sigma_\mu^2(\rho)}$; and second, we construct tests for $H_0^\rho$ that are size-robust to local misspecification in $H_0^{\sigma_\mu^2}$, $LM_{\rho(\sigma_\mu^2)}$ and $C_{\rho(\sigma_\mu^2)}$.

## 4. Monte Carlo results

The baseline model is constructed using $k=2$ in a similar way to Baltagi et al. (2007) with a lattice spatial structure with *rook* spatial dependence among cells. We assume $y_{it}=\alpha+\beta x_{it}+u_{it}$, and we set $\alpha=5$, $\beta=0.5$, and $x_{it}=0.1t+0.5x_{it-1}+z_{it}$ where $z_{it}\sim$Uniform$(-0.5,0.5)$ and $x_{i0}=5+10z_{i0}$. We also set $\sigma_e^2=1$ and we consider different values for $\tau$, $\sigma_\mu^2$ and $\rho$. The spatial autoregressive process is simulated by applying the transformation $y\leftarrow(I_{NT}-\tau W_\otimes)^{-1}y$.

We consider three different panel sizes: $(N=25, T=10)$, $(N=25, T=20)$ and $(N=49, T=10)$. The lattice structure is of size $5\times5$ for the first two cases, and $7\times7$ for the latter. The tables are based on a nominal size of 0.05. For each sample size and parameter settings, we consider 5000 repetitions of the experiment. For each replication, the model was estimated under both ML and IV.[5]

Table 1 reports empirical rejection rates for the marginal test for random effects, $H_0^{\sigma_\mu^2}$. The first set of rows show the unadjusted LM and $C(\alpha)$ statistics, and the last set of rows report the Bera and Yoon (1993) LM and $C(\alpha)$ size-robust statistics. As in Bera et al. (2001), unadjusted statistics show correct 5% rejection rates only when there is no serial correlation, but excessive rejection rates in the presence of serial correlation. The size distortions are larger for the $N=49$, $T=10$ panel size and they do not vary in terms of the value of the spatial autoregressive parameter, $\tau$. Note that the rejection rates reduce considerably for $\rho=0.8$.[6] Size-robust statistics have a better size performance, reducing the size distortion by about a half.

In all cases rejection rates increase as $\sigma_\mu^2$ increases. When the IV estimator is used, $C(\alpha)$ tests have higher rejection rates than LM tests, and the power gain is larger for the $\tau=0.9$ spatial autoregressive coefficient value. Finally, by comparing the unadjusted and size-robust tests we observe no significant power losses ('robustification costs') in the Bera and Yoon (1993) statistics.[7]

Table 2 reports the Monte Carlo results marginal test for serial correlation, $H_0^\rho$. Unadjusted tests are distorted only when large misspecification in the random effects parameter is used ($\sigma_\mu^2\geq0.4$). Moreover, a great size improvement can be observed when the locally size-robust tests are used. The empirical performance of the Bera–Yoon tests determines that they may be applied in non-local settings. That is, in many cases, their robustness properties are maintained for large departures from $H_0^{\sigma_\mu^2}$. In this case, $LM_{\rho(\sigma_\mu^2)}$ has a better performance than $C_{\rho(\sigma_\mu^2)}$ for both ML and IV estimators. In terms of

[4] The LM test is locally most powerful (Cox and Hinkley, 1974).

[5] The simulation exercises were realized using the `spdep` package in `R`, which provides a command for ML estimation under $H_0^{\sigma_\mu^2\rho}$: `lagsarlm`. IV estimation is done as an external function using Two-Stages Least Squares. I am in debt to Nancy Lozano for providing me the `R` codes.

[6] Although not reported in the present version, there is also a considerable drop in power for $\rho=0.8$. The same effect is also found in Baltagi et al. (2007) where they found a significant drop in rejection rates. These authors provide the following explanation for this effect, which fully applies here: "This may be due to the interaction effect between the serial correlation over time due to the AR(1) process on the remainder disturbances and the constant serial correlation over time due to the same region effect." (p. 19) This drop in rejection rates is not as big for $N=25$, $T=20$.

[7] Bera and Yoon (1993) show that their adjusted statistic is sub-optimal when there is no misspecification, i.e. $\rho=0$. See also Bera et al. (2001).

**Table 1**
Tests for $H_0^{\sigma_\mu^2}$.

| $\tau$ | | 0.5 | | | | | | | | | 0.9 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $N=25, T=10$ | | | $N=25, T=20$ | | | $N=49, T=10$ | | | $N=25, T=10$ | | | $N=25, T=20$ | | | $N=49, T=10$ | | |
| Test | | $LM_{\sigma_\mu^2}$ | | $C_{\sigma_\mu^2}$ | $LM_{\sigma_\mu^2}$ | | $C_{\sigma_\mu^2}$ | $LM_{\sigma_\mu^2}$ | | $C_{\sigma_\mu^2}$ | $LM_{\sigma_\mu^2}$ | | $C_{\sigma_\mu^2}$ | $LM_{\sigma_\mu^2}$ | | $C_{\sigma_\mu^2}$ | $LM_{\sigma_\mu^2}$ | | $C_{\sigma_\mu^2}$ |
| Estimator | | ML | IV | IV | ML | IV | IV | ML | IV | IV | ML | IV | IV | ML | IV | IV | ML | IV | IV |
| $\rho$ | $\sigma_\mu^2$ | Empirical size | | | | | | | | | | | | | | | | | |
| 0 | 0 | 0.032 | 0.050 | 0.059 | 0.026 | 0.038 | 0.036 | 0.051 | 0.054 | 0.057 | 0.047 | 0.064 | 0.076 | 0.047 | 0.058 | 0.048 | 0.044 | 0.056 | 0.061 |
| 0.2 | 0 | 0.325 | 0.300 | 0.309 | 0.346 | 0.332 | 0.342 | 0.556 | 0.535 | 0.545 | 0.319 | 0.286 | 0.352 | 0.333 | 0.308 | 0.331 | 0.555 | 0.515 | 0.569 |
| 0.4 | 0 | 0.862 | 0.830 | 0.832 | 0.846 | 0.835 | 0.839 | 0.983 | 0.976 | 0.979 | 0.867 | 0.807 | 0.840 | 0.863 | 0.836 | 0.866 | 0.986 | 0.975 | 0.981 |
| 0.6 | 0 | 0.994 | 0.917 | 0.917 | 0.999 | 0.999 | 0.998 | 1.000 | 0.985 | 0.984 | 0.998 | 0.898 | 0.900 | 0.993 | 0.991 | 0.992 | 1.000 | 0.950 | 0.950 |
| 0.8 | 0 | 0.064 | 0.047 | 0.047 | 0.930 | 0.860 | 0.860 | 0.010 | 0.007 | 0.007 | 0.042 | 0.073 | 0.073 | 0.840 | 0.789 | 0.789 | 0.008 | 0.017 | 0.017 |
| $\rho$ | $\sigma_\mu^2$ | Empirical power | | | | | | | | | | | | | | | | | |
| 0 | 0.2 | 0.203 | 0.196 | 0.209 | 0.576 | 0.555 | 0.566 | 0.326 | 0.322 | 0.332 | 0.200 | 0.198 | 0.240 | 0.553 | 0.510 | 0.542 | 0.331 | 0.305 | 0.381 |
| 0 | 0.4 | 0.852 | 0.800 | 0.808 | 0.991 | 0.991 | 0.991 | 0.991 | 0.985 | 0.986 | 0.858 | 0.793 | 0.826 | 0.996 | 0.995 | 0.996 | 0.988 | 0.982 | 0.984 |
| 0 | 0.6 | 0.999 | 0.961 | 0.960 | 1.000 | 1.000 | 1.000 | 1.000 | 0.995 | 0.995 | 0.990 | 0.942 | 0.946 | 1.000 | 1.000 | 1.000 | 1.000 | 0.984 | 0.984 |
| 0 | 0.8 | 1.000 | 0.952 | 0.951 | 1.000 | 1.000 | 1.000 | 1.000 | 0.994 | 0.994 | 1.000 | 0.918 | 0.918 | 1.000 | 0.998 | 0.998 | 1.000 | 0.960 | 0.960 |
| Test | | $LM_{\sigma_\mu^2(\rho)}$ | | $C_{\sigma_\mu^2(\rho)}$ | $LM_{\sigma_\mu^2(\rho)}$ | | $C_{\sigma_\mu^2(\rho)}$ | $LM_{\sigma_\mu^2(\rho)}$ | | $C_{\sigma_\mu^2(\rho)}$ | $LM_{\sigma_\mu^2(\rho)}$ | | $C_{\sigma_\mu^2(\rho)}$ | $LM_{\sigma_\mu^2(\rho)}$ | | $C_{\sigma_\mu^2(\rho)}$ | $LM_{\sigma_\mu^2(\rho)}$ | | $C_{\sigma_\mu^2(\rho)}$ |
| Estimator | | ML | IV | IV | ML | IV | IV | ML | IV | IV | ML | IV | IV | ML | IV | IV | ML | IV | IV |
| $\rho$ | $\sigma_\mu^2$ | Empirical size | | | | | | | | | | | | | | | | | |
| 0 | 0 | 0.043 | 0.059 | 0.074 | 0.034 | 0.042 | 0.043 | 0.061 | 0.069 | 0.077 | 0.054 | 0.084 | 0.087 | 0.052 | 0.063 | 0.060 | 0.048 | 0.073 | 0.075 |
| 0.2 | 0 | 0.133 | 0.166 | 0.162 | 0.140 | 0.156 | 0.154 | 0.177 | 0.186 | 0.192 | 0.158 | 0.172 | 0.184 | 0.164 | 0.177 | 0.170 | 0.176 | 0.197 | 0.206 |
| 0.4 | 0 | 0.436 | 0.433 | 0.442 | 0.421 | 0.421 | 0.420 | 0.568 | 0.570 | 0.565 | 0.424 | 0.397 | 0.436 | 0.427 | 0.424 | 0.431 | 0.563 | 0.533 | 0.578 |
| 0.6 | 0 | 0.881 | 0.779 | 0.772 | 0.849 | 0.842 | 0.849 | 0.978 | 0.948 | 0.943 | 0.894 | 0.754 | 0.772 | 0.849 | 0.834 | 0.852 | 0.989 | 0.909 | 0.914 |
| 0.8 | 0 | 0.016 | 0.015 | 0.015 | 0.863 | 0.794 | 0.793 | 0.001 | 0.002 | 0.002 | 0.019 | 0.032 | 0.032 | 0.744 | 0.713 | 0.713 | 0.001 | 0.003 | 0.003 |
| $\rho$ | $\sigma_\mu^2$ | Empirical power | | | | | | | | | | | | | | | | | |
| 0 | 0.2 | 0.188 | 0.180 | 0.200 | 0.546 | 0.526 | 0.536 | 0.279 | 0.283 | 0.292 | 0.173 | 0.174 | 0.216 | 0.524 | 0.479 | 0.519 | 0.283 | 0.264 | 0.334 |
| 0 | 0.4 | 0.806 | 0.764 | 0.765 | 0.988 | 0.985 | 0.989 | 0.981 | 0.972 | 0.970 | 0.808 | 0.748 | 0.793 | 0.996 | 0.993 | 0.995 | 0.982 | 0.965 | 0.973 |
| 0 | 0.6 | 0.994 | 0.952 | 0.951 | 1.000 | 1.000 | 1.000 | 1.000 | 0.995 | 0.995 | 0.988 | 0.929 | 0.934 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 0.977 |
| 0 | 0.8 | 0.999 | 0.940 | 0.939 | 1.000 | 1.000 | 1.000 | 1.000 | 0.989 | 0.989 | 0.998 | 0.903 | 0.903 | 1.000 | 0.998 | 0.998 | 1.000 | 0.944 | 0.944 |

Notes: Monte Carlo experiments based on 5000 replications of the baseline model. Theoretical size is 5%. $LM_{\sigma_\mu^2}$ and $C_{\sigma_\mu^2}$: marginal LM and $C(\alpha)$ tests for $H_0^{\sigma_\mu^2}$ assuming $H_0^\rho$. $LM_{\sigma_\mu^2(\rho)}$ and $C_{\sigma_\mu^2(\rho)}$: marginal LM and $C(\alpha)$ Bera and Yoon (1993) locally size-robust tests for $H_0^{\sigma_\mu^2}$, controlling for local departures from $H_0^\rho$. ML: maximum likelihood. IV: Kelejian and Prucha (1997) estimator using $W\mathbf{X}$ as IV.

**Table 2**
Tests for $H_0^{\rho}$.

| $\tau$ | | 0.5 | | | | | | | | | 0.9 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $N=25, T=10$ | | | $N=25, T=20$ | | | $N=49, T=10$ | | | $N=25, T=10$ | | | $N=25, T=20$ | | | $N=49, T=10$ | | |
| Test | | $LM_{\rho}$ | | $C_{\rho}$ | $LM_{\rho}$ | | $C_{\rho}$ | $LM_{\rho}$ | | $C_{\rho}$ | $LM_{\rho}$ | | $C_{\rho}$ | $LM_{\rho}$ | | $C_{\rho}$ | $LM_{\rho}$ | | $C_{\rho}$ |
| Estimator | | ML | IV | IV | ML | IV | IV | ML | IV | IV | ML | IV | IV | ML | IV | IV | ML | IV | IV |
| $\rho$ | $\sigma_{\mu}^2$ | Empirical size | | | | | | | | | | | | | | | | | |
| 0 | 0 | 0.040 | 0.062 | 0.062 | 0.047 | 0.049 | 0.049 | 0.043 | 0.051 | 0.051 | 0.050 | 0.067 | 0.067 | 0.049 | 0.058 | 0.058 | 0.043 | 0.052 | 0.052 |
| 0 | 0.2 | 0.064 | 0.077 | 0.077 | 0.131 | 0.141 | 0.141 | 0.120 | 0.122 | 0.122 | 0.075 | 0.101 | 0.101 | 0.116 | 0.117 | 0.117 | 0.086 | 0.105 | 0.105 |
| 0 | 0.4 | 0.430 | 0.429 | 0.429 | 0.716 | 0.717 | 0.717 | 0.722 | 0.700 | 0.700 | 0.435 | 0.432 | 0.432 | 0.720 | 0.708 | 0.708 | 0.719 | 0.699 | 0.699 |
| 0 | 0.6 | 0.876 | 0.859 | 0.859 | 0.981 | 0.980 | 0.980 | 0.994 | 0.991 | 0.991 | 0.885 | 0.846 | 0.846 | 0.990 | 0.984 | 0.984 | 0.989 | 0.973 | 0.973 |
| 0 | 0.8 | 0.992 | 0.985 | 0.985 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.988 | 0.955 | 0.955 | 1.000 | 1.000 | 1.000 | 1.000 | 0.998 | 0.998 |
| $\rho$ | $\sigma_{\mu}^2$ | Empirical power | | | | | | | | | | | | | | | | | |
| 0.2 | 0 | 0.842 | 0.818 | 0.818 | 0.991 | 0.989 | 0.989 | 0.989 | 0.987 | 0.987 | 0.840 | 0.796 | 0.796 | 0.992 | 0.991 | 0.991 | 0.989 | 0.983 | 0.983 |
| 0.4 | 0 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.997 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.6 | 0 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.998 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.8 | 0 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Test | | $LM_{\rho(\sigma_{\mu}^2)}$ | | $C_{\rho(\sigma_{\mu}^2)}$ | $LM_{\rho(\sigma_{\mu}^2)}$ | | $C_{\rho(\sigma_{\mu}^2)}$ | $LM_{\rho(\sigma_{\mu}^2)}$ | | $C_{\rho(\sigma_{\mu}^2)}$ | $LM_{\rho(\sigma_{\mu}^2)}$ | | $C_{\rho(\sigma_{\mu}^2)}$ | $LM_{\rho(\sigma_{\mu}^2)}$ | | $C_{\rho(\sigma_{\mu}^2)}$ | $LM_{\rho(\sigma_{\mu}^2)}$ | | $C_{\rho(\sigma_{\mu}^2)}$ |
| Estimator | | ML | IV | IV | ML | IV | IV | ML | IV | IV | ML | IV | IV | ML | IV | IV | ML | IV | IV |
| $\rho$ | $\sigma_{\mu}^2$ | Empirical size | | | | | | | | | | | | | | | | | |
| 0 | 0 | 0.035 | 0.063 | 0.063 | 0.051 | 0.056 | 0.055 | 0.043 | 0.048 | 0.053 | 0.049 | 0.083 | 0.092 | 0.050 | 0.064 | 0.064 | 0.064 | 0.078 | 0.086 |
| 0 | 0.2 | 0.031 | 0.042 | 0.048 | 0.047 | 0.048 | 0.048 | 0.034 | 0.043 | 0.043 | 0.039 | 0.083 | 0.100 | 0.035 | 0.049 | 0.053 | 0.032 | 0.061 | 0.084 |
| 0 | 0.4 | 0.035 | 0.066 | 0.066 | 0.025 | 0.022 | 0.021 | 0.030 | 0.049 | 0.052 | 0.026 | 0.072 | 0.103 | 0.032 | 0.041 | 0.046 | 0.031 | 0.069 | 0.098 |
| 0 | 0.6 | 0.057 | 0.146 | 0.141 | 0.021 | 0.028 | 0.033 | 0.134 | 0.199 | 0.207 | 0.093 | 0.165 | 0.189 | 0.021 | 0.040 | 0.055 | 0.131 | 0.193 | 0.234 |
| 0 | 0.8 | 0.430 | 0.501 | 0.484 | 0.119 | 0.160 | 0.158 | 0.644 | 0.697 | 0.679 | 0.480 | 0.483 | 0.505 | 0.187 | 0.206 | 0.215 | 0.719 | 0.695 | 0.698 |
| $\rho$ | $\sigma_{\mu}^2$ | Empirical power | | | | | | | | | | | | | | | | | |
| 0.2 | 0 | 0.717 | 0.709 | 0.709 | 0.986 | 0.981 | 0.980 | 0.932 | 0.926 | 0.923 | 0.725 | 0.702 | 0.669 | 0.980 | 0.978 | 0.972 | 0.955 | 0.948 | 0.924 |
| 0.4 | 0 | 0.996 | 0.978 | 0.980 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.997 | 0.981 | 0.962 | 1.000 | 1.000 | 1.000 | 1.000 | 0.993 | 0.987 |
| 0.6 | 0 | 0.910 | 0.883 | 0.887 | 1.000 | 1.000 | 1.000 | 0.967 | 0.936 | 0.934 | 0.899 | 0.887 | 0.871 | 1.000 | 1.000 | 0.998 | 0.965 | 0.920 | 0.899 |
| 0.8 | 0 | 0.950 | 0.967 | 0.966 | 0.791 | 0.818 | 0.807 | 0.991 | 0.995 | 0.995 | 0.977 | 0.956 | 0.955 | 0.789 | 0.825 | 0.827 | 0.993 | 0.986 | 0.986 |

Notes: Monte Carlo experiments based on 5000 replications of the baseline model. Theoretical size is 5%. $LM_{\rho}$ and $C_{\rho}$: marginal LM and $C(\alpha)$ tests for $H_0^{\rho}$ assuming $H_0^{\sigma_{\mu}^2}$. $LM_{\rho(\sigma_{\mu}^2)}$ and $C_{\rho(\sigma_{\mu}^2)}$: marginal LM and $C(\alpha)$ Bera and Yoon (1993) locally size-robust tests for $H_0^{\rho}$, controlling for local departures from $H_0^{\sigma_{\mu}^2}$. ML: maximum likelihood. IV: Kelejian and Prucha (1997) estimator using $W\mathbf{X}$ as IV.

**Table 3**
Joint tests for $H_0^{\sigma_\mu^2\rho}$.

| $\tau$ | | 0.5 | | | | | | | | | 0.9 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $N=25$, $T=10$ | | | $N=25$, $T=20$ | | | $N=49$, $T=10$ | | | $N=25$, $T=10$ | | | $N=25$, $T=20$ | | | $N=49$, $T=10$ | | |
| Test | | $LM_{\sigma_\mu^2\rho}$ | | $C_{\sigma_\mu^2\rho}$ | $LM_{\sigma_\mu^2\rho}$ | | $C_{\sigma_\mu^2\rho}$ | $LM_{\sigma_\mu^2\rho}$ | | $C_{\sigma_\mu^2\rho}$ | $LM_{\sigma_\mu^2\rho}$ | | $C_{\sigma_\mu^2\rho}$ | $LM_{\sigma_\mu^2\rho}$ | | $C_{\sigma_\mu^2\rho}$ | $LM_{\sigma_\mu^2\rho}$ | | $C_{\sigma_\mu^2\rho}$ |
| Estimator | | ML | IV | IV | ML | IV | IV | ML | IV | IV | ML | IV | IV | ML | IV | IV | ML | IV | IV |
| $\rho$ | $\sigma_\mu^2$ | Empirical size | | | | | | | | | | | | | | | | | |
| 0 | 0 | 0.03 | 0.053 | 0.065 | 0.039 | 0.049 | 0.046 | 0.052 | 0.063 | 0.069 | 0.043 | 0.086 | 0.093 | 0.059 | 0.075 | 0.072 | 0.046 | 0.066 | 0.073 |
| $\rho$ | $\sigma_\mu^2$ | Empirical power | | | | | | | | | | | | | | | | | |
| 0 | 0.2 | 0.163 | 0.165 | 0.178 | 0.498 | 0.499 | 0.503 | 0.256 | 0.268 | 0.281 | 0.169 | 0.181 | 0.220 | 0.477 | 0.458 | 0.492 | 0.265 | 0.265 | 0.322 |
| 0 | 0.4 | 0.791 | 0.776 | 0.778 | 0.989 | 0.987 | 0.988 | 0.981 | 0.976 | 0.976 | 0.812 | 0.768 | 0.799 | 0.994 | 0.992 | 0.993 | 0.984 | 0.977 | 0.979 |
| 0 | 0.6 | 0.996 | 0.966 | 0.970 | 1.000 | 1.000 | 1.000 | 1.000 | 0.995 | 0.996 | 0.989 | 0.944 | 0.938 | 1.000 | 1.000 | 1.000 | 1.000 | 0.980 | 0.978 |
| 0 | 0.8 | 1.000 | 0.952 | 0.961 | 1.000 | 1.000 | 1.000 | 1.000 | 0.990 | 0.991 | 0.998 | 0.925 | 0.915 | 1.000 | 0.998 | 0.998 | 1.000 | 0.952 | 0.951 |
| 0.2 | 0 | 0.769 | 0.757 | 0.758 | 0.981 | 0.977 | 0.977 | 0.973 | 0.971 | 0.972 | 0.761 | 0.741 | 0.737 | 0.976 | 0.977 | 0.978 | 0.979 | 0.970 | 0.966 |
| 0.2 | 0.2 | 0.871 | 0.845 | 0.846 | 0.999 | 0.999 | 0.999 | 0.999 | 0.997 | 0.996 | 0.885 | 0.852 | 0.851 | 0.996 | 0.995 | 0.994 | 0.993 | 0.986 | 0.983 |
| 0.2 | 0.4 | 0.988 | 0.975 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.998 | 0.992 | 0.954 | 0.947 | 1.000 | 1.000 | 1.000 | 1.000 | 0.993 | 0.991 |
| 0.2 | 0.6 | 1.000 | 0.988 | 0.990 | 1.000 | 1.000 | 1.000 | 1.000 | 0.994 | 0.997 | 1.000 | 0.968 | 0.953 | 1.000 | 0.999 | 0.999 | 1.000 | 0.980 | 0.973 |
| 0.2 | 0.8 | 1.000 | 0.961 | 0.969 | 1.000 | 1.000 | 1.000 | 1.000 | 0.988 | 0.990 | 0.997 | 0.937 | 0.920 | 1.000 | 0.998 | 0.998 | 1.000 | 0.969 | 0.965 |
| 0.4 | 0 | 1.000 | 0.996 | 0.996 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.988 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 0.998 | 0.991 |
| 0.4 | 0.2 | 1.000 | 0.998 | 0.995 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 1.000 | 0.987 | 0.965 | 1.000 | 1.000 | 1.000 | 1.000 | 0.996 | 0.993 |
| 0.4 | 0.4 | 1.000 | 0.987 | 0.989 | 1.000 | 1.000 | 1.000 | 1.000 | 0.998 | 1.000 | 1.000 | 0.986 | 0.956 | 1.000 | 1.000 | 1.000 | 1.000 | 0.988 | 0.972 |
| 0.4 | 0.6 | 1.000 | 0.985 | 0.983 | 1.000 | 1.000 | 1.000 | 1.000 | 0.993 | 0.996 | 1.000 | 0.967 | 0.922 | 1.000 | 1.000 | 0.999 | 1.000 | 0.983 | 0.975 |
| 0.4 | 0.8 | 0.997 | 0.935 | 0.943 | 1.000 | 1.000 | 1.000 | 0.999 | 0.972 | 0.977 | 0.991 | 0.939 | 0.896 | 1.000 | 0.999 | 0.999 | 0.998 | 0.957 | 0.941 |
| 0.6 | 0 | 1.000 | 0.983 | 0.987 | 1.000 | 1.000 | 1.000 | 1.000 | 0.987 | 0.990 | 0.999 | 0.973 | 0.927 | 1.000 | 1.000 | 1.000 | 1.000 | 0.978 | 0.946 |
| 0.6 | 0.2 | 1.000 | 0.978 | 0.983 | 1.000 | 1.000 | 1.000 | 1.000 | 0.986 | 0.989 | 0.998 | 0.974 | 0.934 | 1.000 | 0.999 | 0.998 | 0.999 | 0.975 | 0.943 |
| 0.6 | 0.4 | 0.995 | 0.969 | 0.971 | 1.000 | 1.000 | 1.000 | 1.000 | 0.975 | 0.979 | 0.992 | 0.950 | 0.894 | 1.000 | 1.000 | 0.998 | 1.000 | 0.954 | 0.911 |
| 0.6 | 0.6 | 0.952 | 0.900 | 0.905 | 1.000 | 0.999 | 1.000 | 0.991 | 0.924 | 0.932 | 0.914 | 0.898 | 0.834 | 1.000 | 0.995 | 0.994 | 0.970 | 0.919 | 0.881 |
| 0.6 | 0.8 | 0.833 | 0.785 | 0.798 | 1.000 | 1.000 | 1.000 | 0.886 | 0.795 | 0.800 | 0.732 | 0.785 | 0.720 | 1.000 | 0.984 | 0.980 | 0.799 | 0.766 | 0.728 |
| 0.8 | 0 | 0.556 | 0.695 | 0.745 | 0.864 | 0.811 | 0.810 | 0.586 | 0.690 | 0.739 | 0.718 | 0.687 | 0.689 | 0.747 | 0.769 | 0.730 | 0.735 | 0.674 | 0.659 |
| 0.8 | 0.2 | 0.570 | 0.680 | 0.739 | 0.826 | 0.777 | 0.778 | 0.635 | 0.706 | 0.756 | 0.713 | 0.672 | 0.674 | 0.751 | 0.754 | 0.713 | 0.770 | 0.675 | 0.667 |
| 0.8 | 0.4 | 0.589 | 0.697 | 0.749 | 0.807 | 0.733 | 0.735 | 0.696 | 0.767 | 0.807 | 0.761 | 0.707 | 0.718 | 0.661 | 0.714 | 0.667 | 0.808 | 0.682 | 0.690 |
| 0.8 | 0.6 | 0.686 | 0.782 | 0.811 | 0.672 | 0.616 | 0.618 | 0.734 | 0.790 | 0.833 | 0.805 | 0.752 | 0.763 | 0.537 | 0.608 | 0.578 | 0.852 | 0.743 | 0.743 |
| 0.8 | 0.8 | 0.719 | 0.803 | 0.841 | 0.518 | 0.457 | 0.459 | 0.814 | 0.853 | 0.884 | 0.841 | 0.774 | 0.783 | 0.365 | 0.460 | 0.417 | 0.891 | 0.806 | 0.814 |

Notes: Monte Carlo experiments based on 5000 replications of the baseline model. Theoretical size is 5%. $LM_{\sigma_\mu^2\rho}$ and $C_{\sigma_\mu^2\rho}$: joint LM and $C(\alpha)$ tests for $H_0^{\sigma_\mu^2\rho}$. ML: maximum likelihood. IV: Kelejian and Prucha (1997) estimator using $W\mathbf{X}$ as IV.

empirical power, the tests show increasing rejection rates as $\rho$ increases in all except the $N=25$, $T=20$ panel size. For the latter, the rejection rates drop when $\rho=0.8$ is used (see footnote 6).

Table 3 shows that the joint tests have good empirical size, as can be seen by the first row in the table. In general, *LM* tests have a better size performance than $C(\alpha)$ tests. Moreover, all the tests are very responsive to departures from the null hypothesis. Overall we do not observed considerable differences between the ML and IV estimators in terms of the size and power performance of the tests. Small differences appear when comparing the *LM* and $C(\alpha)$ tests in the case where the IV estimator is used and $\tau=0.9$, but the differences do not justify the computation of the $C(\alpha)$ test if the *LM* statistic is available.

## 5. Conclusions and future research

This paper considers a spatial autoregressive panel data model and tests for the presence of random effects and serial autocorrelation within each spatial unit. Ignoring the presence of random effects not only leads to misleading inference but to inconsistent estimation of the regression coefficients. The paper derives joint, marginal and locally size-robust LM and $C(\alpha)$ tests for ML and IV estimators. Monte Carlo results show that standard LM statistics have a good size and power performance for both estimators under correct specification, but they over-reject when the unconsidered parameter is misspecified. In the latter case, locally size-robust statistics bring the empirical rejection rates close to the nominal level. $C(\alpha)$ statistics perform slightly better when the IV estimator is used.

A natural extension of this paper is to test for the presence of spatial autoregressive correlation in panel data models and where the random effects and/or serial correlation parameters are estimated. Another should include both types of spatial dependence as in Anselin and Bera (1998), through a spatial lag of the dependent variable and of the error term.

## Acknowledgments

I am in debt to Anil K. Bera, N. Balakrishnan and an anonymous referee for valuable comments and suggestions. All remaining errors are mine.

## Appendix A

The score functions evaluated at the joint null hypothesis $H_0^{\sigma_\mu^2\rho}$ are

$$s_\beta\Big|_{H_0^{\sigma_\mu^2\rho}}=\frac{1}{\sigma_e^2}X'u,\quad s_\tau\Big|_{H_0^{\sigma_\mu^2\rho}}=-T\sum_i\frac{\psi_i}{1-\tau\psi_i}+\frac{1}{\sigma_e^2}(W_\otimes y)'u,$$

$$s_{\sigma_e^2}\Big|_{H_0^{\sigma_\mu^2\rho}}=-\frac{NT}{2\sigma_e^2}+\frac{1}{2\sigma_e^4}u'u,\quad s_{\sigma_\mu^2}\Big|_{H_0^{\sigma_\mu^2\rho}}=-\frac{NT}{2\sigma_e^2}+\frac{1}{2\sigma_e^4}u'[\iota_T\iota_T'\otimes I_N]u,$$

$$s_\rho\Big|_{H_0^{\sigma_\mu^2\rho}}=\frac{1}{2\sigma_e^2}u'[G_T\otimes I_N]u,$$

where $G_T$ is a $T\times T$ bidiagonal matrix (i.e. $G_T=\{g_{th}\}$, $g_{th}=1$ if $t=h\pm1$, $g_{th}=0$ otherwise), and we are applying the useful result that $\ln|B_\otimes(\tau)|=T\ln|B(\tau)|=T\sum_i\ln(1-\tau\psi_i)$ where $\psi$ denote the eigenvalues of $W$ (Ord, 1975).

The Jacobian is also evaluated at the joint null hypothesis and this gives us

$$J_{\beta\beta}=\frac{\mathbf{X}'\mathbf{X}}{\sigma_e^2},\quad J_{\beta\tau}=\frac{\mathbf{X}'W_\otimes B_\otimes^{-1}(\tau)\mathbf{X}\beta}{\sigma_e^2},\quad J_{\beta\sigma_e^2}=0,\quad J_{\beta\sigma_\mu^2}=0,\quad J_{\beta\rho}=0,$$

$$J_{\tau\tau}=T(\mathrm{tr}[WB(\tau)^{-1}]^2+\mathrm{tr}[(B(\tau)^{-1})'W'WB(\tau)^{-1}])+\frac{(W_\otimes B_\otimes^{-1}(\tau)\mathbf{X}\beta)'W_\otimes B_\otimes^{-1}(\tau)\mathbf{X}\beta}{\sigma_e^2},$$

$$J_{\tau\sigma_e^2}=\frac{T\,\mathrm{tr}[WB(\tau)^{-1}]}{\sigma_e^2},\quad J_{\tau\sigma_\mu^2}=\frac{T\,\mathrm{tr}[WB(\tau)^{-1})]}{\sigma_e^2},\quad J_{\tau\rho}=0,$$

$$J_{\sigma_e^2\sigma_e^2}=\frac{NT}{2\sigma_e^4},\quad J_{\sigma_e^2\sigma_\mu^2}=\frac{NT}{2\sigma_e^4},\quad J_{\sigma_e^2\rho}=0,\quad J_{\sigma_\mu^2\sigma_\mu^2}=\frac{NT^2}{2\sigma_e^4},\quad J_{\sigma_\mu^2\rho}=\frac{N(T-1)}{\sigma_e^2},\quad J_{\rho\rho}=N(T-1).$$

## Appendix B

The LM test for $H_0^{\sigma_\mu^2\rho}$ is $LM_{\sigma_\mu^2\rho} = (s_{\sigma_\mu^2}, s_\rho)\mathbf{J}^{-1}_{\sigma_\mu^2\rho\cdot\theta}(s_{\sigma_\mu^2}, s_\rho)'$. Here $\mathbf{J}_{\sigma_\mu^2\rho\cdot\theta} = \mathbf{J}_{\sigma_\mu^2\rho} - \mathbf{J}_{\sigma_\mu^2\rho,\theta}\mathbf{J}_\theta^{-1}\mathbf{J}_{\theta,\rho\sigma_\mu^2}$ and $\mathbf{J}_{ab,c}$ is the sub-matrix of elements of $\mathbf{J}$ formed by $(ab)$ and $c$, and $\theta, \ = \ \beta, \tau, \sigma_{e^2}$.

Marginal tests for $H_0^{\sigma_\mu^2}$ assuming $H_0^\rho$ and $H_0^\rho$ assuming $H_0^{\sigma_\mu^2}$ can be constructed by using the corresponding score function and by deleting the row and column of the unconsidered parameter ($\rho$ in the first case, $\sigma_\mu^2$ in the second) in $\mathbf{J}$. Let $\mathbf{J}_{-a}$ denote the information matrix where the row and column that corresponds to $a$ were deleted. Then, we have $LM_{\sigma_\mu^2} = s^2_{\sigma_\mu^2}\mathbf{J}^{-1}_{-\rho:\sigma_\mu^2\cdot\theta}$ and $LM_\rho = s_\rho^2\mathbf{J}^{-1}_{-\sigma_\mu^2:\rho\cdot\theta} = s_\rho^2\mathbf{J}_\rho^{-1}$.

The extension of Bera et al. (2001) locally size-robust tests to our SAR model is given by the following test statistics: $LM_{\sigma_\mu^2(\rho)} = s^2_{\sigma_\mu^2(\rho)}\mathbf{J}^{-1}_{\sigma_\mu^2(\rho)\cdot\theta}$ and $LM_{\rho(\sigma_\mu^2)} = s^2_{\rho(\sigma_\mu^2)}\mathbf{J}^{-1}_{\rho(\sigma_\mu^2)\cdot\theta}$. In this case, $s_{\sigma_\mu^2(\rho)} = s_{\sigma_\mu^2} - \mathbf{J}_{\sigma_\mu^2\rho\cdot\theta}\mathbf{J}^{-1}_{\rho\cdot\theta}s_\rho = s_{\sigma_\mu^2} - \mathbf{J}_{\sigma_\mu^2\rho\cdot\theta}\mathbf{J}_\rho^{-1}s_\rho$, $s_{\rho(\sigma_\mu^2)} = s_\rho - \mathbf{J}_{\rho\sigma_\mu^2\cdot\theta}\mathbf{J}^{-1}_{\sigma_\mu^2\cdot\theta}s_{\sigma_\mu^2}$, $\mathbf{J}_{\sigma_\mu^2(\rho)\cdot\theta} = \mathbf{J}_{\sigma_\mu^2\cdot\theta} - \mathbf{J}_{\sigma_\mu^2\rho\cdot\theta}\mathbf{J}^{-1}_{\rho\cdot\theta}\mathbf{J}_{\rho\sigma_\mu^2\cdot\theta} = \mathbf{J}_{\sigma_\mu^2\cdot\theta} - \mathbf{J}_{\sigma_\mu^2\rho\cdot\theta}\mathbf{J}_\rho^{-1}\mathbf{J}_{\rho\sigma_\mu^2\cdot\theta}$, and $\mathbf{J}_{\rho(\sigma_\mu^2)\cdot\theta} = \mathbf{J}_{\rho\cdot\theta} - \mathbf{J}_{\rho\sigma_\mu^2\cdot\theta}\mathbf{J}^{-1}_{\sigma_\mu^2\cdot\theta}\mathbf{J}_{\sigma_\mu^2\rho\cdot\theta} = \mathbf{J}_\rho - \mathbf{J}_{\rho\sigma_\mu^2\cdot\theta}\mathbf{J}^{-1}_{\sigma_\mu^2\cdot\theta}\mathbf{J}_{\sigma_\mu^2\rho\cdot\theta}$. Note that $LM_{\rho(\sigma_\mu^2)} \neq LM_\rho$ due to the fact that the elements in the Jacobian matrix corresponding to $\rho$ and $\sigma_\mu^2$ are not zero.

For the IV estimation method, $C(\alpha)$ tests can be constructed for the joint and individual hypotheses as follows: $C_{\sigma_\mu^2\rho} = (s_{\sigma_\mu^2\cdot\theta}, s_\rho)\mathbf{J}^{-1}_{\sigma_\mu^2\rho\cdot\theta}(s_{\sigma_\mu^2\cdot\theta}, s_\rho)$, $C_{\sigma_\mu^2} = s^2_{\sigma_\mu^2\cdot\theta}\mathbf{J}^{-1}_{-\rho:\sigma_\mu^2\cdot\theta}$ and $C_\rho = s^2_{\rho\cdot\theta}\mathbf{J}^{-1}_{-\sigma_\mu^2:\rho\cdot\theta}$. In this case, $s_{\sigma_\mu^2\cdot\theta} = s_{\sigma_\mu^2} - \mathbf{J}_{-\rho:\sigma_\mu^2,\theta}\mathbf{J}^{-1}_{-\rho:\theta}s_\theta$ and $s_{\rho\cdot\theta} = s_\rho - \mathbf{J}_{-\sigma_\mu^2:\rho,\theta}\mathbf{J}^{-1}_{-\sigma_\mu^2:\theta}s_\theta = s_\rho$ are the so-called effective scores. Note that $C_\rho = LM_\rho$.

Finally, locally size-robust $C(\alpha)$ tests for the IV estimator are defined as $C_{\sigma_\mu^2(\rho)} = s^2_{\sigma_\mu^2(\rho)\cdot\theta}\mathbf{J}^{-1}_{\sigma_\mu^2(\rho)\cdot\theta}$ and $C_{\rho(\sigma_\mu^2)} = s^2_{\rho(\sigma_\mu^2)\cdot\theta}\mathbf{J}^{-1}_{\rho(\sigma_\mu^2)\cdot\theta}$. In this case, $s_{\sigma_\mu^2(\rho)\cdot\theta} = s_{\sigma_\mu^2\cdot\theta} - \mathbf{J}_{\sigma_\mu^2\rho\cdot\theta}\mathbf{J}^{-1}_{\rho\cdot\theta}s_{\rho\cdot\theta} = s_{\sigma_\mu^2\cdot\theta} - \mathbf{J}_{\sigma_\mu^2\rho\cdot\theta}\mathbf{J}_\rho^{-1}s_\rho$, $s_{\rho(\sigma_\mu^2)\cdot\theta} = s_{\rho\cdot\theta} - \mathbf{J}_{\rho\sigma_\mu^2\cdot\theta}\mathbf{J}^{-1}_{\sigma_\mu^2\cdot\theta}s_{\sigma_\mu^2\cdot\theta} = s_\rho - \mathbf{J}_{\rho\sigma_\mu^2\cdot\theta}\mathbf{J}^{-1}_{\sigma_\mu^2\cdot\theta}s_{\sigma_\mu^2\cdot\theta}$, $\mathbf{J}_{\sigma_\mu^2(\rho)\cdot\theta} = \mathbf{J}_{\sigma_\mu^2\cdot\theta} - \mathbf{J}_{\sigma_\mu^2\rho\cdot\theta}\mathbf{J}^{-1}_{\rho\cdot\theta}\mathbf{J}_{\rho\sigma_\mu^2\cdot\theta} = \mathbf{J}_{\sigma_\mu^2\cdot\theta} - \mathbf{J}_{\sigma_\mu^2\rho\cdot\theta}\mathbf{J}_\rho^{-1}\mathbf{J}_{\rho\sigma_\mu^2\cdot\theta}$, and $\mathbf{J}_{\rho(\sigma_\mu^2)\cdot\theta} = \mathbf{J}_{\rho\cdot\theta} - \mathbf{J}_{\rho\sigma_\mu^2\cdot\theta}\mathbf{J}^{-1}_{\sigma_\mu^2\cdot\theta}\mathbf{J}_{\sigma_\mu^2\rho\cdot\theta} = \mathbf{J}_\rho - \mathbf{J}_{\rho\sigma_\mu^2\cdot\theta}\mathbf{J}^{-1}_{\sigma_\mu^2\cdot\theta}\mathbf{J}_{\sigma_\mu^2\rho\cdot\theta}$.

## References


Anselin, L., Bera, A.K., 1998. Spatial dependence in linear regression models with an introduction to spatial econometrics. In: Ullah, A., Giles, D.E.A. (Eds.), Handbook of Applied Econometrics. Marcel Dekker, New York, pp. 237–290.

Baltagi, B.H., Song, S.H., Koh, W., 2003. Testing panel data regression models with spatial error correlation. Journal of Econometrics 68, 133–151.

Baltagi, B.H., Song, S.H., Koh, W., Jung, B.C., 2007. Testing for serial correlation, spatial autocorrelation and random effects using panel data. Journal of Econometrics 140, 5–51.

Bera, A.K., Bilias, Y., 2001. Rao's score, Neyman C(α) and Silvey's LM 18 tests: an essay on historical developments and some new results. Journal of Statistical Planning and Inference 97, 9–44.

Bera, A.K., Sosa-Escudero, W., Yoon, M.J., 2001. Tests for the error component model in the presence of local misspecification. Journal of Econometrics 101, 1–23.

Bera, A.K., Yoon, M.J., 1993. Specification testing with locally misspecified alternatives. Econometric Theory 9, 649–658.

Breusch, T., Pagan, A., 1980. The Lagrange multiplier test and its applications to model specification in econometrics. Review of Economic Studies 47, 239–253.

Cox, D.R., Hinkley, D.V., 1974. Theoretical Statistics. Chapman & Hall, CRC Press, New York.

Elhorst, J.P., 2003. Specification and estimation of spatial panel data models. International Regional Science Review 26, 244–268.

Kelejian, H.H., Prucha, I.R., 1997. Estimation of the spatial autoregressive parameter by two stage least squares procedures: a serious problem. International Regional Science Review 20, 103–111.

Ord, K., 1975. Estimation methods for models of spatial interaction. Journal of the American Statistical Association 70, 120–126.