Online Appendix

Tests of Asset Pricing with Time-Varying Factor Loads

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This document contains three sections. Appendix A collects the mathematical proofs of the results presented in the paper. Appendix B contains an extension of our tests for exact asset pricing factor models. In Appendix C we develop a detailed Monte-Carlo exercise showing the finite-sample performance of the tests. The numbers of propositions, lemmas and equations refer to those presented in the main paper.

Appendix A: Proofs of the Main Results

In this paper we use the same set of assumptions as in Ando and Bai (2015). In particular, assumptions A–F below are direct adaptations of the conditions in Ando and Bai (2015) for a panel data model with cross-sectional correlation.

In the second set of conditions, we impose assumptions on the generated regressors. The dynamic factor loadings β_{it} are estimated by the autoregressive model (3.7)

$$\omega_{(i,j),t+1} = \delta_{ij,0} + \delta_{ij,1}\omega_{(i,j),t} + v_{ij,t+1},$$

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such that $\mathrm{E}_t[\omega_{(i,j),t+1}] = \delta_{ij,0} + \delta_{ij,1}\omega_{(i,j),t}$. Then, $\widehat{\beta}_{ij,t} = \widehat{\mathrm{E}}_t[\omega_{(i,j),t+1}] = \widehat{\delta}_{ij,0} + \widehat{\delta}_{ij,1}\omega_{(i,j),t}$, with $(\widehat{\delta}_{ij,0}, \widehat{\delta}_{ij,1})$ the OLS parameter estimators of the above time series regression equation. Note that $\{\omega_{(i,j),t}\}_{t=1}^{T}$ are themselves generated from the procedure detailed in Section 3.1, and they depend on $m \to \infty$. As such β_{it} depends on two layers of generated regressors. Assumptions GR1–GR3 concern, respectively, the convergence of the realized covariance estimates in the first step by employing the sequential asymptotics, the generated regressors errors, and a generated regressors covariance matrix.

Consider the following assumptions.

Assumption A. (Common Factors). The common factors satisfy $\mathrm{E}\|\widetilde{g}_t\|^4 < \infty$. Furthermore, $T^{-1}\sum_{t=1}^{T} \widetilde{g}_t\widetilde{g}_t^\top \to \Sigma_G$ as $T \to \infty$, where Σ_G is an $R \times R$ positive definite matrix.

Assumption B. (Factor Loadings). The factor-loading matrix $\beta^* = [\beta_1^*, ..., \beta_N^*]^\top$ satisfies $\mathrm{E}\|(\beta_i^*)^4\| < \infty$ and $\|N^{-1}\beta^{*\top}\beta^* - \Sigma_{\beta^*}\| \to 0$ as $N \to \infty$, where Σ_{β^*} is an $R \times R$ positive definite matrix.

Assumption C. (Error Terms). There exists a positive constant $C < \infty$ such that for all N and T,

1. $\mathrm{E}[\varepsilon_{i,t}] = 0$, $\mathrm{E}[|\varepsilon_{i,t}|^8] < C$ for all i and t;
2. $\varepsilon_{i,t}$ and $\varepsilon_{j,s}$ are independent, for $i \neq j$ and $t \neq s$.
3. For every (s,t), $\mathrm{E}[|N^{-1/2}\sum_{i=1}^{N}(\varepsilon_{i,s}\varepsilon_{i,t} - \mathrm{E}[\varepsilon_{i,s}\varepsilon_{i,t}])|^4] < C$.
4. $\varepsilon_{i,t}$ is independent of X_{js}, β_i^*, and $\widetilde{g}_s$ for all i, j, t, s.

Assumption D. (Explanatory Variables). We assume $\mathrm{E}\|X_{it}\|^4 < C$. The $p \times p$ matrix $\frac{1}{T}[X_i^\top M_{G^0} X_i]$ is positive definite, where $M_G = I - G(G^\top G)^{-1}G$, and M_{G^0} is equal to M_G evaluated at the true common factors G^0. Furthermore, we define $A_i = \frac{1}{T}X_i^\top M_G X_i$, $B_i = (\beta_i^*\beta_i^{*\top}) \otimes I_T$, $C_i = \frac{1}{\sqrt{T}}\beta_i^{*\top} \otimes (X_i^\top M_G)$. Let $\mathcal{A}$ be the collection of G such that $\mathcal{A} = \{G :$

$G^\top G/T = I\}$. we assume that

$$\inf_{G\in\mathcal{A}}\left[\frac{1}{N}\sum_{i=1}^{N}\mathrm{E}_i(G)\right] \quad \text{is positive definite}$$

where $\mathrm{E}_i(G) = B_i - C_i^\top A_i^- C_i$ and A_i^- is the generalized inverse of A_i.

Assumption E. (Central Limit Theory). Let $H_i := X_i^\top M_G \varepsilon_i - (\eta_i \odot M_G X_i)^\top X_i (X_i^\top X_i)^{-1}(X_i \odot v_i)$. As T goes to infinity,

$$W_i^{-1/2}\frac{1}{\sqrt{T}}H_i \xrightarrow{d} N(0, I),$$

where W_i is defined in equation (3.15).

Assumption F. (JointLimit). Let $\zeta_i = \frac{1}{\sqrt{T}}X_i^\top M_{G^0}\varepsilon_i$, $\zeta = (\zeta_1^\top, \zeta_2^\top, ..., \zeta_N^\top)^\top$, and $W =$ block-diag$(W_1, W_2, ..., W_N)$, where W_i is defined in Assumption E. We assume, as $N, T \to \infty$,

$$\frac{\zeta^\top W^{-1}\zeta - N(K+1)}{\sqrt{2N(K+1)}} \to N(0, 1).$$

Assumption GR1. (Sequential Asymptotics). For each pair (i, j), let $\widehat{Z}_{ij} = (1, \omega_{(i,j)})$ be the realized covariance estimates and $Z_{ij} = (1, \mathrm{QC}_{(i,j)})$ the population quadratic covariance measures defined in (3.4). Assume that as $(m, T, N)_{seq} \to \infty$, we have that

$$\widehat{\Omega}_{t+1} \xrightarrow{p} \mathrm{QC}_{t+1}, \text{ as } m \to \infty.$$

In addition, the OLS parameter estimators satisfy

$$\sqrt{T}(\widehat{\delta}_{ij} - \delta_{ij}) = \left(\frac{\widehat{Z}_{ij}^\top \widehat{Z}_{ij}}{T}\right)^{-1}\frac{\widehat{Z}_{ij}^\top v_{ij}}{\sqrt{T}} = \left(\frac{Z_{ij}^\top Z_{ij}}{T}\right)^{-1}\frac{Z_{ij}^\top v_{ij}}{\sqrt{T}} + o_p(1),$$

and

$$\sqrt{T}(\widehat{\beta}_{ij} - \beta_{ij}) = \widehat{Z}_{ij}\left(\frac{\widehat{Z}_{ij}^\top \widehat{Z}_{ij}}{T}\right)^{-1}\frac{\widehat{Z}_{ij}^\top v_{ij}}{\sqrt{T}} = Z_{ij}\left(\frac{Z_{ij}^\top Z_{ij}}{T}\right)^{-1}\frac{Z_{ij}^\top v_{ij}}{\sqrt{T}} + o_p(1).$$

Assumption GR2. (Errors in GR). For each pair (i,j), v_{ij} satisfy the same moment conditions in Assumption C, and ε_i and v_{ij} are mutually independent and independent of Z_{ij}.

Assumption GR3. Let H_i be a $T \times K$ matrix with columns defined by the vectors $Z_{ij}\left(\frac{Z_{ij}^\top Z_{ij}}{T}\right)^{-1}\frac{Z_{ij}^\top v_{ij}}{\sqrt{T}}$ for $j = 1, \ldots, K$. Then, for all i, $\left(\lambda_i \odot \frac{M_G X_i}{T}\right)^\top H_i$ has probability limit that is bounded as $T \to \infty$.

In what follows we establish the results discussed in the main text.

Proof of Lemma 3.1. The proof of this lemma follows from an application of generated regressors in Pagan (1984) and Wooldridge (2012, appendix to ch.9) to the results in Song (2013) and Ando and Bai (2015).

Let $r_i^e = (r_{i,2}^e, \ldots, r_{i,T}^e)^\top$ be the dependent variable, for the ith time series regression model discussed in (2.10)

$$r_{i,t+1}^e = \widehat{X}_{it}\eta_i + \nu_{i,t+1},$$
$$\nu_{i,t+1} = \beta_i^{*\top}\widetilde{g}_{t+1} + \varepsilon_{i,t+1},$$

where $\eta_i = (\alpha_i, \lambda_i^\top)^\top$ denote the model parameters for $i = 1, \ldots, N$, and $\widehat{X}_{it} = (1, \widehat{\beta}_{it}^\top)$ with $\widehat{\beta}_{it}$, which will be denoted as the generated regressors, that converge to $X_{it} = (1, \beta_{it}^\top)$, as $T \to \infty$, with $\beta_{it} = (\beta_{i1,t}, \ldots, \beta_{iK,t})^\top$ such that $\beta_{ij,t} = \mathrm{E}_t[\omega_{(i,j),t+1}]$ with $\omega_{(i,j),t+1}$ elements of the vector $\widehat{\Omega}_{i,t+1}^{ur}$ defined in Section 3.1.

From OLS algebra we have that

$$\widehat{\eta}_i = (\widehat{X}_i^\top M_{\widehat{G}}\widehat{X}_i)^{-1}\widehat{X}_i^\top M_{\widehat{G}} r_i^e,$$

with $M_{\widehat{G}}$ the residual projection matrix of the regressors on the unobserved factors.

We operate in matrix form for each asset in the cross-section $i = 1, \ldots, N$. Let's write $r_i^e = X_i \eta_i + (\widehat{X}_i - X_i)\eta_i + G\beta_i^* + \varepsilon_i$. The matrices X_i and $\widehat{X}_i$ share a column of ones corresponding to the intercept parameter. Then, the excess return on the risky assets can be also written as $r_i^e = X_i\eta_i + (\widehat{\beta}_i - \beta_i)\lambda_i + G\beta_i^* + \varepsilon_i$ where, abusing of notation, $\beta_i = (\beta_{i1}, \ldots, \beta_{iK})$ and $\widehat{\beta}_i = (\widehat{\beta}_{i1}, \ldots, \widehat{\beta}_{iK})$ are $T \times K$ matrices.

After some algebra with the OLS estimator, we obtain

$$\sqrt{T}\left(\widehat{\eta}_i - \eta_i\right) = \left(\frac{\widehat{X}_i^\top M_{\widehat{G}} \widehat{X}_i}{T}\right)^{-1} \frac{\widehat{X}_i^\top M_{\widehat{G}}}{T} \left[\sqrt{T}\left(\widehat{\beta}_i - \beta_i\right)\lambda_i + \sqrt{T} G\beta_i^* + \sqrt{T}\varepsilon_i\right] + o_p(1),$$

where we are using the fact that $\sqrt{T}\left(\widehat{\eta}_i - \left(\frac{\widehat{X}_i^\top M_{\widehat{G}} \widehat{X}_i}{T}\right)^{-1} \frac{\widehat{X}_i^\top M_{\widehat{G}} X_i}{T}\eta_i\right) = \sqrt{T}\left(\widehat{\eta}_i - \eta_i\right) + o_p(1)$, as $T \to \infty$. Assumption GR1 determines that by using the sequential asymptotics structure, as $m \to \infty$ diverges to infinity first, we will only consider the effect of the second layer of the generated regressors structure, i.e., the effect produced by using $\widehat{X}_i$ instead of X_i. In particular, this expression shows that the additional effect in the parameter estimators due to using generated regressors is through the difference between $\widehat{\beta}_i$ and β_i. From Assumption GR1 and GR2,

$$\sqrt{T}\left(\widehat{\eta}_i - \eta_i\right) = \left(\frac{\widehat{X}_i^\top M_{\widehat{G}} \widehat{X}_i}{T}\right)^{-1} \frac{\widehat{X}_i^\top M_{\widehat{G}}}{T} \left[H_i\lambda_i + \sqrt{T} G\beta_i^* + \sqrt{T}\varepsilon_i\right] + o_p(1).$$

For the first term, we can write the expression as

$$\frac{\widehat{X}_i^\top M_{\widehat{G}}}{T} H_i \lambda_i = \left(\lambda_i \odot \frac{M_{\widehat{G}} \widehat{X}_i}{T}\right)^\top H_i.$$

We study now the second term. By Proposition 1 in Song (2013), this term can be written as

$$\frac{1}{\sqrt{T}} \widehat{X}_i^\top M_{\widehat{G}} G\beta_i^* = \frac{1}{N}\sum_{s=1}^{N} L_{is}(\widehat{\eta}_s - \eta_s) + o_p(1),$$

where $L_{is} = a_{is}(X_i^\top M_G X_s)/T$ with $a_{is} = (\beta_i^*)^\top (G^\top G/N)^{-1}\beta_s^*$. Thus,

$$\sqrt{T}(\widehat{\eta}_i - \eta_i) = \widehat{S}_i^{-1}\left[\left(\lambda_i \odot \frac{M_{\widehat{G}}\widehat{X}_i}{T}\right)^\top H_i + \frac{1}{N}\sum_{s=1}^{N} L_{is}\sqrt{T}(\widehat{\eta}_s - \eta_s) + \frac{\widehat{X}_i^\top M_{\widehat{G}}\varepsilon_i}{\sqrt{T}}\right] + o_p(1).$$

Noting that $S_i = \frac{X_i^\top M_{\widehat{G}} X_i}{T} = \frac{\widehat{X}_i^\top M_{\widehat{G}} \widehat{X}_i}{T} + o_p(1)$, we obtain

$$\sqrt{T}(\widehat{\eta}_i - \eta_i) = S_i^{-1}\left[\left(\lambda_i \odot \frac{M_{\widehat{G}}\widehat{X}_i}{T}\right)^\top H_i + \frac{1}{N}\sum_{s=1}^{N} L_{is}\sqrt{T}(\widehat{\eta}_s - \eta_s) + \frac{\widehat{X}_i' M_{\widehat{G}}\varepsilon_i}{\sqrt{T}}\right] + o_p(1).$$

Note however, that we are interested in the variance of the entire vector $\widehat{\eta}_\cdot = (\widehat{\eta}_1, \ldots, \widehat{\eta}_N)^\top$. The above equation implies, stacking over i,

$$\sqrt{T}(\widehat{\eta}_\cdot - \eta_\cdot) = S^{-1}\left[\left(\lambda \odot \frac{M_{\widehat{G}}\widehat{X}}{T}\right)^\top H + \frac{1}{N} L\sqrt{T}(\widehat{\eta} - \eta) + \frac{\widehat{X}^\top M_{\widehat{G}}\varepsilon}{\sqrt{T}}\right] + o_p(1),$$

with H and S block-diagonal matrices with elements H_i and S_i. Then,

$$\left(S - \frac{1}{N}L\right)\sqrt{T}(\widehat{\eta}_\cdot - \eta_\cdot) = \left(\lambda \odot \frac{M_{\widehat{G}}\widehat{X}}{T}\right)^\top H + \frac{\widehat{X}^\top M_{\widehat{G}}\varepsilon}{\sqrt{T}} + o_p(1),$$

and

$$\mathrm{Var}\left(\sqrt{T}(\widehat{\eta}_\cdot - \eta_\cdot)\right) = \left[\left(S - \frac{1}{N}L^\top\right)\right]^{-1} \mathrm{Var}\left(\left(\lambda \odot \frac{M_{\widehat{G}}\widehat{X}}{T}\right)^\top H + \frac{\widehat{X}^\top M_{\widehat{G}}\varepsilon}{\sqrt{T}}\right)\left[\left(S - \frac{1}{N}L\right)\right]^{-1} + o_p(1).$$

Under the assumption imposing a zero covariance between the errors v_{ij} and ε_i for all i, j, the asymptotic variance of the quantity $\sqrt{T}\,(\widehat{\eta}_\cdot - \eta_\cdot)$ is

$$\left[\left(S - \frac{1}{N}L^\top\right)\right]^{-1} W \left[\left(S - \frac{1}{N}L\right)\right]^{-1},$$

as $(m, N, T)_{seq} \to \infty$, with W a $N(K+1) \times N(K+1)$ block-diagonal matrix given by

$$W = \lim_{T\to\infty} \mathrm{Var}\left[\left(\lambda \odot \frac{M_G X}{T}\right)^\top H\right] + \lim_{T\to\infty} \mathrm{Var}\left(\frac{X^\top M_G \varepsilon}{\sqrt{T}}\right),$$

where we are also using the fact that $\frac{M_{\widehat{G}}\widehat{X}}{T} = \frac{M_G X}{T} + o_p(1)$ and $\frac{\widehat{X}^\top M_{\widehat{G}}\widehat{X}}{T} = \frac{X^\top M_G X}{T} + o_p(1)$, as $(m, N, T)_{seq} \to \infty$. Further algebra shows that

$$W = \lim_{T\to\infty} \mathrm{Var}\left[\left(\lambda \odot \frac{M_G X}{T}\right)^\top H\right] + \lim_{T\to\infty} \left(\frac{X^\top M_G X}{T}\right) Var(\varepsilon).$$

□

Proof of Lemma 3.2. In addition to assumptions in Lemma 3.1, we need two additional assumptions E and F. In this scenario the result immediately follows from the consistency results in Song (2013), and the properties of the OLS estimators $\widehat{\delta}_{ij} = (\widehat{\delta}_{ij,0}, \widehat{\delta}_{ij,1})^\top$ in regression (3.7). □

Proof of Proposition 3.1. To show this result, we follow closely the results for Theorem 1 in Ando and Bai (2015). We require the same set of assumptions (A–F) and the additional assumptions on the generated regressors (GR1–GR3). Under these conditions, it follows that

$$\sqrt{T}\left(\widehat{\eta}_{\cdot} - \eta_{\cdot}\right)^\top \left[\left(S - \frac{1}{N}L^\top\right)\right] W^{-1} \left[\left(S - \frac{1}{N}L\right)\right] \sqrt{T}\left(\widehat{\eta}_{\cdot} - \eta_{\cdot}\right)$$

converges to a chi-square distribution with $(N-1)K+N$ degrees of freedom, as $(m, N, T)_{seq} \to \infty$. The standardized version of the test statistic, given by

$$\frac{T\left(\widehat{\eta}_{\cdot} - \eta_{\cdot})\right)^\top \left[\left(S - \frac{1}{N}L^\top\right)\right] W^{-1} \left[\left(S - \frac{1}{N}L\right)\right] \left(\widehat{\eta}_{\cdot} - \eta_{\cdot}\right) - \left[(N-1)K + N\right]}{2\sqrt{(N-1)K + N}},$$

converges to a $N(0, 1)$ with $\sqrt{T}/N \to 0$. Furthermore, under these assumptions, replacing

η by $\widehat{\eta}$, and the unknown elements in S, L and W by their consistent estimators, does not alter the result. $\square$

Proof of Propositions 3.2 and 3.3. The proof follows from the proof of Proposition 3.1. $\square$

Appendix B: Asset pricing tests without cross-sectional dependence

In the scenario of asset pricing testing without cross-sectional dependence the relevant time series model can be written as

$$r^e_{i,t+1} = \widehat{X}_{it}\eta_i + \varepsilon_{i,t+1},$$

with $\varepsilon_{i,t+1}$ the *i.i.d.* error terms. For the sake of comparison with $\widehat{\Gamma}_{\alpha,\lambda}$ in (3.19), we propose the following test statistic for the null hypothesis $H_0^{\alpha,\lambda}$:

$$\widetilde{\Gamma}_{\alpha,\lambda} = \frac{T\left(\widetilde{\eta}. - \widetilde{\eta}\iota_N\right)^\top \widetilde{S}'\widetilde{V}^{-1}\widetilde{S}\left(\widetilde{\eta}. - \widetilde{\eta}\iota_N\right) - [(N-1)K+N]}{\sqrt{2[(N-1)K+N]}},$$

with $\widetilde{\eta}. = (\widetilde{\eta}_1, \ldots, \widetilde{\eta}_N)^\top$; $\widetilde{\eta}_i$ are the OLS estimators of η_i obtained from the above regression, $\widetilde{\eta}$ is the counterpart of $\widehat{\eta}$ defined after expression (3.18), and $\widetilde{S}^{-1}\widetilde{V}\widetilde{S}^{-1}$ with $\widetilde{S} = (\widehat{X}'\widehat{X})/T$, is a consistent estimator of the asymptotic covariance matrix of the quantity $\sqrt{T}\left(\widetilde{\eta}. - \eta.\right)$ with $\widetilde{\eta}. = (\widetilde{\eta}_1, \ldots, \widetilde{\eta}_N)^\top$ and $\eta. = (\eta_1, \ldots, \eta_N)^\top$. The matrix $\widetilde{V}$ is block-diagonal with elements $\widetilde{V}_i$ defined as

$$\widetilde{V}_i = \left(\widetilde{\lambda}_i \odot \frac{\widehat{X}_i}{T}\right)^\top T^{-1} diag\left(\widehat{H}_i^\top \widehat{H}_i\right)\left(\widetilde{\lambda}_i \odot \frac{\widehat{X}_i}{T}\right) + \left(\frac{\widehat{X}_i^\top \widehat{X}_i}{T}\right)\widetilde{\sigma}_i^2,$$

with $\widehat{H}_i$ defined in Lemma 3.2 and $\widetilde{\sigma}_i^2 = \frac{1}{T-(K+1)}\sum_{t=1}^{T}(r^e_{i,t+1} - \widehat{X}_{it}\widetilde{\eta}_i)^2$.

Under these conditions and further regularity conditions similar to those stated in Proposition 3.1, the limiting distribution of the test statistic $\widetilde{\Gamma}_{\alpha,\lambda}$ under the null hypothesis $H_0^{\alpha,\lambda}$ satisfies

$$\widetilde{\Gamma}_{\alpha,\lambda} \xrightarrow{d} N(0,1), \tag{0.1}$$

as $(m, N, T)_{seq} \to \infty$, for $\sqrt{T}/N \to 0$. The proof of this result is analogous to the proof of Proposition 3.1 but simpler as it does not involve the projection of the generated regressors on the unobserved factors. Moreover, we can use this result to test the null hypotheses H_0^{α} and H_0^{λ}, for which the test statistics will be defined as $\widetilde{\Gamma}_{\alpha}$ and $\widetilde{\Gamma}_{\lambda}$, respectively.

Appendix C: Monte Carlo experiments

This section explores the small sample performance of the proposed tests through a Monte Carlo experiment.

Design

Consider the regression model developed in Section 2, eq. (2.14), that we copy here:

$$r^e_{i,t+1} = \alpha_i + \hat{\beta}_i^\top \lambda_i + \nu_{i,t+1},$$
$$\nu_{i,t+1} = \beta_i^{*\top} \widetilde{g}_{t+1} + \varepsilon_{i,t+1}.$$

For convenience we rewrite the $\nu_{i,t+1}$ error term as

$$\nu_{i,t+1} = g_{i,t+1} + \varepsilon_{i,t+1},$$

where we define $\beta_i^{*\top} \widetilde{g}_{t+1} := g_{i,t+1}$. This data generating process (DGP) allows us to evaluate the presence of heterogeneity in the intercepts (i.e., α_i) and/or slopes (i.e., λ_i), together with (unobserved) factor components on the performance of the above tests. We consider a regression model with two covariates.

Parameter heterogeneity is first analyzed as in Ando and Bai (2015) by using $\lambda_i = (\lambda_{1i}, 1/2)$ as the coefficients, for which we will allow for heterogeneity in the first component only. We consider two cases: (*1*) $\lambda_{1i} = -1/2$ (no slope heterogeneity); (*2*) $\lambda_{1i} \sim i.i.d.\ U(-0.65, -0.45)$ (slope heterogeneity). In addition, as our specific model is concerned with the intercept heterogeneity, we consider two additional possibilities, where we have either (i) $\alpha = 0$ (no intercept heterogeneity); and (ii) $\alpha \sim i.i.d.\ U(0, 1)$ (intercept heterogeneity).

The error term $\nu_{i,t+1}$ has zero mean but it may exhibit cross-sectional correlation for $i = 1, \ldots, N$ due to the presence of the demeaned common factors $g_{i,t+1}$ in the error term. In order to evaluate the effect of cross-sectional correlation we simulate two different error structures: (A) $g_{i,t+1} = 0$ (i.e., no cross-sectional correlation); and another (B) with a 2-factor structure of the form $g_{i,t+1} = \mu_{1i}\delta_{1,t+1} + \mu_{2i}\delta_{2,t+1}$, where μ_{1i}, μ_{2i}, $\delta_{1,t+1}$, $\delta_{2,t+1}$ are $i.i.d.\ N(0, 1)$. Assume also that $\varepsilon_{i,t+1} \sim i.i.d. N(0, 1)$. The error structure of type (B) corresponds to the 2-factor structure of the simulation experiments in Ando and Bai (2015).

Let $\hat{\beta}_i = (\hat{\beta}_{1i}, \hat{\beta}_{2i})$ be a generated regressors structure with two regressors. In order to construct generated regressors we assume that $\beta_{jit} = \beta^*_{jit} + v_{jit}$ for $j = 1, 2$, such that $\widehat{\beta}_{jit}$ is the predicted value of individual regressions of T observations β_{jit} on β^*_{jit}, separately, for each $j = 1, 2$, and $i = 1, \ldots, N$, as in Section 3.1. Note that in this case, $\mathrm{E}[\beta_{jit}] = \mathrm{E}[\beta^*_{jit}] = 0$ and $\mathrm{E}[\widehat{\beta}_{jit}] = 0$. We assume that $v_{jit} \sim i.i.d.\ Unif(-1, 1)$ for $j = 1, 2$, $i = 1, \ldots, N$ and $t = 1, \ldots, T$. The Ando and Bai (2015) factor model allows the unobserved factors to be correlated with the covariates. In our case we consider two different models: (a) $\beta^*_{jit} \sim i.i.d.\ Unif(-2, 2)$ for $j = 1, 2$, $i = 1, \ldots, N$ and $t = 1, \ldots, T$; and (b) $\beta^*_{jit} \sim i.i.d.\ Unif(-2, 2) + g_{i,t+1}$ for $j = 1, 2$, $i = 1, \ldots, N$ and $t = 1, \ldots, T$. In case (b) they are correlated with the unobserved factor structure.

Our interest lies in evaluating the empirical size and power performance of the proposed tests described in the previous section, which account for generated regressors. The test statistics defined above are given by $\widehat{\Gamma}_\alpha$ for $H_0^\alpha : \alpha_i = 0 \quad \forall i$ (intercepts only), $\widehat{\Gamma}_\lambda$ for $H_0^\lambda : \lambda_i = \lambda \;\; \forall i$ (slopes only), and $\widehat{\Gamma}_{\alpha,\lambda}$ for $H_0^{\alpha,\lambda} : \alpha_i = 0 \;\; \& \; \lambda_i = \lambda$ for all i (intercepts and slopes). For completeness, we also provide results for the corresponding versions $\widetilde{\Gamma}_\alpha$, $\widetilde{\Gamma}_\lambda$ and $\widetilde{\Gamma}_{\alpha,\lambda}$ constructed from Pesaran and Yamagata (2008)'s Swamy type test under the presence of generated regressors but assuming that there are no unobserved factors.[1] We also compare these tests against the uncorrected counterparts given by not correcting the tests for the additional uncertainty produced by replacing the unobservable regressors β_{it} by $\widehat{\beta}_{it}$. The uncorrected Ando and Bai (2015) tests are denoted by the superscript AB and the tests corresponding to the uncorrected Pesaran and Yamagata (2008) test are denoted by the superscript PY.

We consider sample sizes similar to those in our empirical application, $N = 50$ and $T = 100, 200$. All simulation exercises are based on $1,000$ replications. We consider 1%, 5%, and 10% nominal sizes in all experiments.

Results

The results for the simulations are presented in Tables A1–A8. Tables are divided in three column panels, with the left panel (columns C1-C4) reporting the rejection rates for the tests for the null intercept hypothesis (Γ_α), the middle panel (columns C5-C8) reporting the rejection rates for the slope homogeneity hypothesis (Γ_λ), and the right panel (columns C9-C12) for the joint tests ($\Gamma_{\alpha,\lambda}$). Each table is divided into two row panels where we consider the cases determined by the taxonomy described above: *1* (no slope heterogeneity), *2* (slope heterogeneity).

Table A1 corresponds to DGPs of type iAa (i.e., no intercept heterogeneity and no factor

[1] See Appendix B for the asymptotic properties of the Pesaran and Yamagata (2008) tests with generated regressors correction only (that is, no cross-sectional dependence is accounted for).

structure): DGP1iAa, DGP2iAa. Table A2 is type *iiAa* (i.e., intercept heterogeneity and no factor structure): DGP1iiAa, DGP2iiAa. Table A3 is type *iBa* (i.e., no intercept heterogeneity and factor structure but covariates are not correlated with the factor structure): DGP1iBa, DGP2iBa. Table A4 is type *iiBa* (i.e., intercept heterogeneity and factor structure but covariates are not correlated with the factor structure): DGP1iiBa, DGP2iiBa. Table A5 is type *iBb* (i.e., no intercept heterogeneity, factor structure and covariates are correlated with the factor structure): DGP1iBb, DGP2iBb. Table A6 is type *iiBb* (i.e., intercept heterogeneity, factor structure and covariates are correlated with the factor structure): DGP1iiBb, DGP2iiBb. Table A7 is type *iBa* (i.e., no intercept heterogeneity, no factor structure in the error term and covariates with a factor structure): DGP1iBa, DGP2iBa. Table A8 is type *iiBa* (i.e., intercept heterogeneity, no factor structure in the error term and covariates with a factor structure): DGP1iiBa, DGP2iiBa.

The results in Tables A1, A3, A5, and A7, columns C1-C4, show that the developed tests for the intercept only (i.e., $\widehat{\Gamma}_\alpha$) have empirical sizes close to the nominal sizes for the tests for the intercepts. Moreover, $\widehat{\Gamma}_\alpha$ has better or equal size performance than $\widetilde{\Gamma}_\alpha$, $\widehat{\Gamma}_\alpha^{AB}$ and $\widetilde{\Gamma}_\alpha^{PY}$. The rejection rates approach the correct size when T increases. The simulation shows that the rejection rates for the intercepts are not affected by parameter heterogeneity in the slopes and thus, they detect this particular type of heterogeneity. Tables A2, A4, A6 and A8, columns C1-C4, show that under the alternative (i.e., when there is heterogeneity in the intercept) the tests correctly reject the null hypothesis.

The results in columns C5-C8 in all tables show that tests for slope heterogeneity have appropriate empirical size under the null hypothesis (when $\lambda_{1i} = -1/2$). In particular, our constructed test statistic $\widehat{\Gamma}_\lambda$ has, in general, the best size performance. In fact, the proposed variance correction for generated regressors seems to work well, in contrast with $\widehat{\Gamma}_\lambda^{AB}$ that exhibits a clear over-rejection under the null hypothesis. The presence of unobserved factors does not invalidate the standard Swamy-type tests (i.e., $\widetilde{\Gamma}_\alpha$) if the factors are not correlated with the covariates (see Tables A3 and A4, DGP1iBa, DGP1iiBa) but they are oversized when the regressors are correlated with the factor structure (see Tables A5 and A6, DGP1iBb, DGP1iiBb). These findings suggest that the presence of unobserved factors that are correlated with the covariates, which is likely to be the case in our empirical set up, invalidate the results of Swamy-type tests even under the correction of the asymptotic variance introduced earlier for generated regressors. Our proposed test, however, has appropriate size in all cases. When taking into account the potential oversize of the other tests, our proposed test has the best power performance too, thus being able to detect departures from the null of slope homogeneity.

For completeness we also evaluate in Tables A7 and A8 the effect of covariates processes with a factor structure even when the error term does not contain a factor structure, and thus they are not correlated with each other. The size and power properties determine that the DGPs of the covariates are not driving the results above, as the empirical size and power are similar to those where there is no factor structure in the error term.

The statistics for the joint hypothesis, $H_0^{\alpha,\lambda}$: $\alpha_i = 0$ & $\lambda_i = \lambda$ for all i (intercepts and slopes, columns C9-C12), follow the performance of the corresponding marginal tests for either intercepts and slopes.

The test results show that: (i) the tests are able to separate heterogeneity arising from the intercepts and the slopes; (ii) in the presence of unobserved factors, Ando and Bai (2015) type tests are the only ones that have correct empirical size; (iii) the presence of generated regressors requires a variance correction that is achieved with our proposed test. Overall, our proposed tests, $\widehat{\Gamma}_\alpha$, $\widehat{\Gamma}_\lambda$ and $\widehat{\Gamma}_{\alpha,\lambda}$, have the best performance in terms of correct empirical size and power for detection of departures from the different null hypotheses.

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Table A1 - Rejection rates. $\alpha_i = 0, \forall i$, **no factor structure** $(g_{it} = 0)$

			Intercept				Slopes				Intercept&Slopes			
Size	N	T	$\widehat{\Gamma}_{\alpha}$	$\widehat{\Gamma}^{AB}_{\alpha}$	$\widetilde{\Gamma}_{\alpha}$	$\widetilde{\Gamma}^{PY}_{\alpha}$	$\widehat{\Gamma}_{\lambda}$	$\widehat{\Gamma}^{AB}_{\lambda}$	$\widetilde{\Gamma}_{\lambda}$	$\widetilde{\Gamma}^{PY}_{\lambda}$	$\widehat{\Gamma}_{\alpha,\lambda}$	$\widehat{\Gamma}^{AB}_{\alpha,\lambda}$	$\widetilde{\Gamma}_{\alpha,\lambda}$	$\widetilde{\Gamma}^{PY}_{\alpha,\lambda}$
			C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
DGP1iAa: , $\lambda_{1i} = -1/2$														
0.01	50	100	0.039	0.038	0.041	0.046	0.016	0.076	0.010	0.037	0.020	0.047	0.011	0.032
0.01	50	200	0.026	0.025	0.026	0.030	0.008	0.032	0.006	0.026	0.009	0.031	0.013	0.024
0.05	50	100	0.100	0.098	0.107	0.112	0.067	0.186	0.035	0.117	0.055	0.145	0.038	0.107
0.05	50	200	0.085	0.084	0.091	0.095	0.027	0.097	0.023	0.078	0.033	0.089	0.033	0.068
0.10	50	100	0.166	0.163	0.170	0.172	0.112	0.267	0.067	0.186	0.1	0.217	0.082	0.164
0.10	50	200	0.138	0.137	0.146	0.151	0.060	0.166	0.044	0.124	0.061	0.144	0.055	0.123
DGP2iAa: $\lambda_{1i} \sim iid\, U(-.65, -.45)$														
0.01	50	100	0.035	0.034	0.040	0.043	0.683	0.862	0.676	0.846	0.547	0.720	0.549	0.710
0.01	50	200	0.023	0.023	0.024	0.027	0.978	0.995	0.983	0.996	0.926	0.971	0.951	0.984
0.05	50	100	0.095	0.097	0.103	0.109	0.837	0.934	0.836	0.925	0.721	0.86	0.715	0.858
0.05	50	200	0.088	0.089	0.093	0.101	0.988	0.990	0.973	0.993	0.985	0.996	1.000	1.000
0.10	50	100	0.164	0.167	0.169	0.170	0.899	0.955	0.888	0.949	0.811	0.905	0.803	0.903
0.10	50	200	0.135	0.137	0.144	0.148	0.998	1.000	0.999	1.000	0.986	0.997	0.993	0.998

Tabla A2 - Rejection rates. $\alpha_i \sim iid\, U(0,1), \forall i$, no factor structure ($g_{it} = 0$)

			Intercept				Slopes				Intercept&Slopes			
Size	N	T	$\widehat{\Gamma}_\alpha$	$\widehat{\Gamma}_\alpha^{AB}$	$\widetilde{\Gamma}_\alpha$	$\widetilde{\Gamma}_\alpha^{PY}$	$\widehat{\Gamma}_\lambda$	$\widehat{\Gamma}_\lambda^{AB}$	$\widetilde{\Gamma}_\lambda$	$\widetilde{\Gamma}_\lambda^{PY}$	$\widehat{\Gamma}_{\alpha,\lambda}$	$\widehat{\Gamma}_{\alpha,\lambda}^{AB}$	$\widetilde{\Gamma}_{\alpha,\lambda}$	$\widetilde{\Gamma}_{\alpha,\lambda}^{PY}$
			C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
DGP1iiAa: $\lambda_{1i} = -1/2$														
0.01	50	100	1.000	1.000	1.000	1.000	0.017	0.074	0.01	0.038	1.000	1.000	1.000	1.000
0.01	50	200	1.000	1.000	1.000	1.000	0.008	0.032	0.004	0.026	1.000	1.000	1.000	1.000
0.05	50	100	1.000	1.000	1.000	1.000	0.065	0.186	0.035	0.117	1.000	1.000	1.000	1.000
0.05	50	200	1.000	1.000	1.000	1.000	0.026	0.097	0.023	0.078	1.000	1.000	1.000	1.000
0.10	50	100	1.000	1.000	1.000	1.000	0.11	0.268	0.064	0.186	1.000	1.000	1.000	1.000
0.10	50	200	1.000	1.000	1.000	1.000	0.059	0.165	0.044	0.124	1.000	1.000	1.000	1.000
DGP2iiAa: $g_{it} = 0$, $\lambda_{1i} \sim iid\, U(-.65, -.45)$														
0.01	50	100	1.000	1.000	1.000	1.000	0.685	0.863	0.676	0.846	1.000	1.000	1.000	1.000
0.01	50	200	1.000	1.000	1.000	1.000	0.977	0.994	0.983	0.996	1.000	1.000	1.000	1.000
0.05	50	100	1.000	1.000	1.000	1.000	0.839	0.933	0.836	0.925	1.000	1.000	1.000	1.000
0.05	50	200	1.000	1.000	1.000	1.000	0.992	1.000	0.996	1.000	1.000	1.000	1.000	1.000
0.10	50	100	1.000	1.000	1.000	1.000	0.898	0.956	0.888	0.949	1.000	1.000	1.000	1.000
0.10	50	200	1.000	1.000	1.000	1.000	0.998	1.000	0.999	1.000	1.000	1.000	1.000	1.000

Table A3 - Rejection rates. $\alpha_i = 0, \forall i$, **factor structure** ($g_{it} = \mu_{1i}\delta_{1t} + \mu_{2i}\delta_{2t}$), **covariates not correlated with factors**

			Intercept				Slopes				Intercept&Slopes			
Size	N	T	$\widehat{\Gamma}_\alpha$	$\widehat{\Gamma}_\alpha^{AB}$	$\widetilde{\Gamma}_\alpha$	$\widetilde{\Gamma}_\alpha^{PY}$	$\widehat{\Gamma}_\lambda$	$\widehat{\Gamma}_\lambda^{AB}$	$\widetilde{\Gamma}_\lambda$	$\widetilde{\Gamma}_\lambda^{PY}$	$\widehat{\Gamma}_{\alpha,\lambda}$	$\widehat{\Gamma}_{\alpha,\lambda}^{AB}$	$\widetilde{\Gamma}_{\alpha,\lambda}$	$\widetilde{\Gamma}_{\alpha,\lambda}^{PY}$
			C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
DGP1iBa: $\lambda_{1i} = -1/2$														
0.01	50	100	0.026	0.027	0.187	0.189	0.001	0.009	0.024	0.046	0.003	0.009	0.117	0.136
0.01	50	200	0.015	0.016	0.177	0.177	0.001	0.01	0.013	0.026	0.004	0.01	0.111	0.124
0.05	50	100	0.075	0.077	0.231	0.231	0.008	0.047	0.069	0.121	0.012	0.04	0.172	0.2
0.05	50	200	0.058	0.060	0.222	0.222	0.008	0.036	0.047	0.078	0.015	0.032	0.163	0.184
0.10	50	100	0.125	0.126	0.269	0.269	0.026	0.087	0.122	0.172	0.028	0.077	0.21	0.236
0.10	50	200	0.101	0.102	0.254	0.254	0.018	0.071	0.084	0.143	0.023	0.063	0.193	0.213
DGP2iBa: $\lambda_{1i} \sim iid\ U(-.65, -.45)$														
0.01	50	100	0.023	0.029	0.189	0.189	0.476	0.695	0.281	0.376	0.316	0.515	0.238	0.275
0.01	50	200	0.014	0.020	0.177	0.177	0.951	0.986	0.646	0.740	0.856	0.941	0.452	0.53
0.05	50	100	0.074	0.079	0.231	0.231	0.678	0.831	0.478	0.579	0.521	0.712	0.348	0.409
0.05	50	200	0.056	0.062	0.222	0.222	0.985	0.998	0.815	0.876	0.942	0.986	0.591	0.669
0.10	50	100	0.121	0.130	0.269	0.269	0.760	0.889	0.591	0.683	0.634	0.796	0.418	0.492
0.10	50	200	0.098	0.101	0.254	0.254	0.993	0.999	0.886	0.926	0.977	0.994	0.680	0.756

Tabla A4 - Rejection rates. $\alpha_i \sim iid\, U(0,1), \forall i$, **factor structure** ($g_{it} = \mu_{1i}\delta_{1t} + \mu_{2i}\delta_{2t}$), **covariates not correlated with factors**

			Intercept				Slopes				Intercept&Slopes			
Size	N	T	$\widehat{\Gamma}_\alpha$	$\widehat{\Gamma}_\alpha^{AB}$	$\widetilde{\Gamma}_\alpha$	$\widetilde{\Gamma}_\alpha^{PY}$	$\widehat{\Gamma}_\lambda$	$\widehat{\Gamma}_\lambda^{AB}$	$\widetilde{\Gamma}_\lambda$	$\widetilde{\Gamma}_\lambda^{PY}$	$\widehat{\Gamma}_{\alpha,\lambda}$	$\widehat{\Gamma}_{\alpha,\lambda}^{AB}$	$\widetilde{\Gamma}_{\alpha,\lambda}$	$\widetilde{\Gamma}_{\alpha,\lambda}^{PY}$
			C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
DGP1iiBa: $\lambda_{1i} = -1/2$														
0.01	50	100	1.000	1.000	1.000	1.000	0.001	0.009	0.024	0.046	1.000	1.000	1.000	1.000
0.01	50	200	1.000	1.000	1.000	1.000	0.001	0.010	0.013	0.026	1.000	1.000	1.000	1.000
0.05	50	100	1.000	1.000	1.000	1.000	0.008	0.047	0.069	0.121	1.000	1.000	1.000	1.000
0.05	50	200	1.000	1.000	1.000	1.000	0.008	0.036	0.047	0.078	1.000	1.000	1.000	1.000
0.10	50	100	1.000	1.000	1.000	1.000	0.026	0.087	0.122	0.172	1.000	1.000	1.000	1.000
0.10	50	200	1.000	1.000	1.000	1.000	0.018	0.071	0.084	0.143	1.000	1.000	1.000	1.000
DGP2iiBa: $\lambda_{1i} \sim iid\, U(-.65,-.45)$														
0.01	50	100	1.000	1.000	1.000	1.000	0.476	0.695	0.281	0.376	1.000	1.000	1.000	1.000
0.01	50	200	1.000	1.000	1.000	1.000	0.951	0.986	0.646	0.740	1.000	1.000	1.000	1.000
0.05	50	100	1.000	1.000	1.000	1.000	0.678	0.831	0.478	0.579	1.000	1.000	1.000	1.000
0.05	50	200	1.000	1.000	1.000	1.000	0.985	0.998	0.815	0.876	1.000	1.000	1.000	1.000
0.10	50	100	1.000	1.000	1.000	1.000	0.760	0.889	0.591	0.683	1.000	1.000	1.000	1.000
0.10	50	200	1.000	1.000	1.000	1.000	0.993	0.999	0.886	0.926	1.000	1.000	1.000	1.000

Table A5 - Rejection rates. $\alpha_i = 0, \forall i$, **factor structure** ($g_{it} = \mu_{1i}\delta_{1t} + \mu_{2i}\delta_{2t}$), **covariates correlated with factors**

			Intercept				Slopes				Intercept&Slopes			
Size	N	T	$\widehat{\Gamma}_\alpha$	$\widehat{\Gamma}_\alpha^{AB}$	$\widetilde{\Gamma}_\alpha$	$\widetilde{\Gamma}_\alpha^{PY}$	$\widehat{\Gamma}_\lambda$	$\widehat{\Gamma}_\lambda^{AB}$	$\widetilde{\Gamma}_\lambda$	$\widetilde{\Gamma}_\lambda^{PY}$	$\widehat{\Gamma}_{\alpha,\lambda}$	$\widehat{\Gamma}_{\alpha,\lambda}^{AB}$	$\widetilde{\Gamma}_{\alpha,\lambda}$	$\widetilde{\Gamma}_{\alpha,\lambda}^{PY}$
			C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
DGP1iBb: $\lambda_{1i} = -1/2$														
0.01	50	100	0.026	0.025	0.079	0.079	0.019	0.050	1.000	1.000	0.017	0.037	1.000	1.000
0.01	50	200	0.014	0.016	0.075	0.075	0.018	0.034	1.000	1.000	0.012	0.019	1.000	1.000
0.05	50	100	0.084	0.086	0.165	0.165	0.075	0.141	1.000	1.000	0.073	0.105	1.000	1.000
0.05	50	200	0.063	0.065	0.157	0.157	0.060	0.104	1.000	1.000	0.047	0.066	1.000	1.000
0.10	50	100	0.125	0.129	0.227	0.227	0.144	0.230	1.000	1.000	0.113	0.166	1.000	1.000
0.10	50	200	0.102	0.103	0.216	0.216	0.100	0.172	1.000	1.000	0.073	0.127	1.000	1.000
DGP2iBb: $\lambda_{1i} \sim iid\, U(-.65, -.45)$														
0.01	50	100	0.026	0.026	0.079	0.079	0.734	0.809	1.000	1.000	0.571	0.662	1.000	1.000
0.01	50	200	0.017	0.017	0.075	0.075	0.988	0.997	1.000	1.000	0.954	0.972	1.000	1.000
0.05	50	100	0.081	0.081	0.165	0.165	0.863	0.911	1.000	1.000	0.748	0.817	1.000	1.000
0.05	50	200	0.062	0.062	0.157	0.157	0.998	1.000	1.000	1.000	0.986	0.995	1.000	1.000
0.10	50	100	0.127	0.127	0.227	0.227	0.910	0.949	1.000	1.000	0.819	0.871	1.000	1.000
0.10	50	200	0.104	0.104	0.216	0.216	1.000	1.000	1.000	1.000	0.995	0.998	1.000	1.000

Tabla A6 - Rejection rates. $\alpha_i \sim iid\, U(0,1), \forall i$, factor structure ($g_{it} = \mu_{1i}\delta_{1t} + \mu_{2i}\delta_{2t}$), covariates correlated with factors

			Intercept				Slopes				Intercept&Slopes			
Size	N	T	$\widehat{\Gamma}_\alpha$	$\widehat{\Gamma}_\alpha^{AB}$	$\widetilde{\Gamma}_\alpha$	$\widetilde{\Gamma}_\alpha^{PY}$	$\widehat{\Gamma}_\lambda$	$\widehat{\Gamma}_\lambda^{AB}$	$\widetilde{\Gamma}_\lambda$	$\widetilde{\Gamma}_\lambda^{PY}$	$\widehat{\Gamma}_{\alpha,\lambda}$	$\widehat{\Gamma}_{\alpha,\lambda}^{AB}$	$\widetilde{\Gamma}_{\alpha,\lambda}$	$\widetilde{\Gamma}_{\alpha,\lambda}^{PY}$
			C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
DGP1iiBb: $\lambda_{1i} = -1/2$														
0.01	50	100	1.000	1.000	1.000	1.000	0.020	0.051	1.000	1.000	1.000	1.000	1.000	1.000
0.01	50	200	1.000	1.000	1.000	1.000	0.019	0.036	1.000	1.000	1.000	1.000	1.000	1.000
0.05	50	100	1.000	1.000	1.000	1.000	0.076	0.144	1.000	1.000	1.000	1.000	1.000	1.000
0.05	50	200	1.000	1.000	1.000	1.000	0.060	0.104	1.000	1.000	1.000	1.000	1.000	1.000
0.10	50	100	1.000	1.000	1.000	1.000	0.143	0.232	1.000	1.000	1.000	1.000	1.000	1.000
0.10	50	200	1.000	1.000	1.000	1.000	0.103	0.174	1.000	1.000	1.000	1.000	1.000	1.000
DGP2iiBb: $\lambda_{1i} \sim iid\, U(-.65, -.45)$														
0.01	50	100	1.000	1.000	1.000	1.000	0.736	0.810	1.000	1.000	1.000	1.000	1.000	1.000
0.01	50	200	1.000	1.000	1.000	1.000	0.988	0.997	1.000	1.000	1.000	1.000	1.000	1.000
0.05	50	100	1.000	1.000	1.000	1.000	0.864	0.913	1.000	1.000	1.000	1.000	1.000	1.000
0.05	50	200	1.000	1.000	1.000	1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.10	50	100	1.000	1.000	1.000	1.000	0.912	0.950	1.000	1.000	1.000	1.000	1.000	1.000
0.10	50	200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table A7 - Rejection rates. $\alpha_i = 0, \forall i$, no factor structure ($g_{it} = 0$), covariates with a factor structure

			Intercept				Slopes				Intercept&Slopes			
Size	N	T	$\widehat{\Gamma}_{\alpha}$	$\widehat{\Gamma}_{\alpha}^{AB}$	$\widetilde{\Gamma}_{\alpha}$	$\widetilde{\Gamma}_{\alpha}^{PY}$	$\widehat{\Gamma}_{\lambda}$	$\widehat{\Gamma}_{\lambda}^{AB}$	$\widetilde{\Gamma}_{\lambda}$	$\widetilde{\Gamma}_{\lambda}^{PY}$	$\widehat{\Gamma}_{\alpha,\lambda}$	$\widehat{\Gamma}_{\alpha,\lambda}^{AB}$	$\widetilde{\Gamma}_{\alpha,\lambda}$	$\widetilde{\Gamma}_{\alpha,\lambda}^{PY}$
			C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
DGP1iBa: $\lambda_{1i} = -1/2$														
0.01	50	100	0.037	0.039	0.043	0.044	0.006	0.058	0.003	0.033	0.007	0.054	0.004	0.038
0.01	50	200	0.026	0.029	0.028	0.031	0.005	0.031	0.005	0.031	0.008	0.033	0.006	0.031
0.05	50	100	0.102	0.104	0.096	0.099	0.025	0.161	0.020	0.116	0.041	0.119	0.035	0.093
0.05	50	200	0.079	0.082	0.086	0.088	0.019	0.089	0.017	0.088	0.028	0.082	0.024	0.082
0.10	50	100	0.156	0.153	0.161	0.162	0.059	0.259	0.034	0.180	0.075	0.197	0.061	0.161
0.10	50	200	0.132	0.135	0.141	0.138	0.031	0.151	0.033	0.130	0.046	0.149	0.042	0.135
DGP2iBa: $\lambda_{1i} \sim iid\, U(-.65, -.45)$														
0.01	50	100	0.038	0.038	0.045	0.048	0.997	0.999	0.999	1.000	0.980	0.995	0.983	0.999
0.01	50	200	0.028	0.029	0.029	0.033	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.05	50	100	0.101	0.102	0.098	0.097	0.999	1.000	1.000	1.000	0.994	1.000	1.000	1.000
0.05	50	200	0.081	0.084	0.087	0.086	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.10	50	100	0.153	0.155	0.159	0.163	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.10	50	200	0.136	0.134	0.141	0.144	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table A8 - Rejection rates. $\alpha_i \sim iid\, U(0,1), \forall i$, **no factor structure ($g_{it} = 0$), covariates with a factor structure**

			Intercept				Slopes				Intercept&Slopes			
Size	N	T	$\widehat{\Gamma}_{\alpha}$	$\widehat{\Gamma}_{\alpha}^{AB}$	$\widetilde{\Gamma}_{\alpha}$	$\widetilde{\Gamma}_{\alpha}^{PY}$	$\widehat{\Gamma}_{\lambda}$	$\widehat{\Gamma}_{\lambda}^{AB}$	$\widetilde{\Gamma}_{\lambda}$	$\widetilde{\Gamma}_{\lambda}^{PY}$	$\widehat{\Gamma}_{\alpha,\lambda}$	$\widehat{\Gamma}_{\alpha,\lambda}^{AB}$	$\widetilde{\Gamma}_{\alpha,\lambda}$	$\widetilde{\Gamma}_{\alpha,\lambda}^{PY}$
			C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
DGP1iiBa: $\lambda_{1i} = -1/2$														
0.01	50	100	1.000	1.000	1.000	1.000	0.006	0.058	0.003	0.033	1.000	1.000	1.000	1.000
0.01	50	200	1.000	1.000	1.000	1.000	0.005	0.031	0.005	0.031	1.000	1.000	1.000	1.000
0.05	50	100	1.000	1.000	1.000	1.000	0.025	0.161	0.020	0.116	1.000	1.000	1.000	1.000
0.05	50	200	1.000	1.000	1.000	1.000	0.019	0.089	0.017	0.088	1.000	1.000	1.000	1.000
0.10	50	100	1.000	1.000	1.000	1.000	0.059	0.259	0.034	0.180	1.000	1.000	1.000	1.000
0.10	50	200	1.000	1.000	1.000	1.000	0.031	0.151	0.033	0.130	1.000	1.000	1.000	1.000
DGP2iiBa: $\lambda_{1i} \sim iid\, U(-.65, -.45)$														
0.01	50	100	1.000	1.000	1.000	1.000	0.997	0.999	0.999	1.000	1.000	1.000	1.000	1.000
0.01	50	200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.05	50	100	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.05	50	200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.10	50	100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.10	50	200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000